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FITTING A DISTRIBUTION TO CATASTROPHIC EVENT

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FITTING A DISTRIBUTION TO CATASTROPHIC EVENT

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science at Virginia Commonwealth University.

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Abstract

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By Ebenezer Kwadwo Osei

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science at Virginia Commonwealth University.

Virginia Commonwealth University, 2011

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Statistics is a branch of mathematics which is heavily employed in the area of Actuarial Mathematics. This thesis first reviews the importance of statistical distributions in the analysis of insurance problems and the applications of Statistics in the area of risk and insurance. The Normal, Log-normal, Pareto, Gamma, standard Beta, Frechet, Gumbel, Weibull, Poisson, binomial, and negative binomial distributions are looked at and the importance of these distributions in general insurance is also emphasized.

A careful review of literature is to provide practitioners in the general insurance industry with statistical tools which are of immediate application in the industry. These tools include estimation methods and fit statistics popular in the insurance industry. Finally this thesis carries out the task of fitting statistical distributions to the flood loss data in the 50 States of the United States.

Chapter 1: Introduction

1.1 Background

The need for risk management has become necessary because without it, it is very difficult to deal with losses resulting from events like Hurricane Katrina that hit the gulf coast in 2005, the New Orleans flooding and the Tsunami which occurred in the Indian Ocean. O'Connor *et al* (2004) explains that human societies worldwide have always experienced floods, creating a prominent role for floods in legends, religions, and history. Undeniably, many more types of these floods of this magnitude have occurred but have not yet been studied or reported.

Floods are among the most powerful forces on earth and have played an important role in the past as far as shaping our world is concerned. But the recognition of the important role that floods play in shaping our cultural and physical landscape also owes to increased understanding of the variety of mechanisms that cause floods and how the types and magnitudes of floods can vary with time and space. Floods come about as a result of too much rainfall, snow or a mixture of high river levels and high tides. But Thompson (1964) indicated that most of the largest documented floods resulted from breaches of other types of natural dams, including landslide dams, ice dams from smaller glaciers, releases from caldera lakes and ice-jam floods. The emergence of these natural disasters has left questions about who is in charge of the economic and human losses of these catastrophes.

It is in this regard that the government and insurance companies (insurers) have come together in the past to take away the suffering of such disasters by establishing the Federal Emergency Management Agency (FEMA) in 1979. The work of FEMA came about through the congressional act of 1803 to provide disaster relief on the federal level after a fire destroyed a New Hampshire town (Federal Emergency Management Agency). In 1968 the National Flood Insurance Program (NFIP) was started by the National Flood Insurance Agency (NFIA). FEMA operates under the auspices of the Department of Homeland Security (DHS) since becoming part of DHS in 2003. This agency has helped promote economic growth and development of the country in the wake of such catastrophes.

This thesis carries out the task of fitting distributions to loss data. The economic effect of this statistical exercise is manifested in determination of insurance premiums. In the following section I discuss briefly how insurance premiums can be determined following statistical analysis.

1.2 How to Calculate an Insurance Premium

Policyholders buy insurance to obtain security from risk. Having analyzed a random risk *S*, an insurance company will want to decide how much it should charge to handle (take responsibility for) the risk, and whether or not it should set aside reserves in case of extreme or unlikely event occurring. These problems have to be considered in the light of a very competitive market for insurance. Given a risk exposure *S*, we refer to its expected loss value *E(S)* as the Actuarially Fair Premium (AFP) for the risk (Mas-Colel *et al* 1995 and Kleiber *et al* 2003). Clearly, an insurance company must charge more than the premium to cover expenses, allow for variability in the number and amount of claims, and make a profit. In setting the premium rates, the general insurer must take account of the relevant risk factors and behavior of the policyholders with respect of these risk factors. In determining a premium for a risk, one accounts for variability in all these tailors and also take into account administrative cost to obtain the gross premium. Actually, administrative costs are clearly important in practice, and changes in policy details (like the introduction of deductible) often influence both claim and administrative costs. In order to quote a premium for a product, insurance companies need to predict future frequency and severity of claims. Hossack *et al* (1983) explained that calculating the premium must take into account claim frequency and average claim size. I present a scenario on how to calculate flood premium based on the expected value principle (Boland 2007).

Let's assume that premiums vary by state, since some states clearly are more likely to have flood events than others.

Assume, for a given year for a given state:

- f is the probability of a flood event that qualifies for a claim (obtained from statistical analysis of frequency of claims).
- n is the number of insured entities in the state.
- c is expected claim amount for a single insured entity given a qualifying event (obtained from statistical analysis of severity of losses).
- C is total expected claim amount, fnc (For this study, C was provided and I wish to find the probability distribution of C)

O is the operational cost to run the insurance company in the state. This would include staffing (salary and benefits), real estate, taxes, legal, and incidental cost of running the business etc.

- P is the premium per insured entity.
- T is the total cost (claim plus operational costs, $C+O$)
- R is the total premium revenue, nP

· M is the target profit margin, i.e. the fraction that the Revenue exceeds the Total costs

M = $\frac{R}{T} - 1$

$$nP = R = (M+1)T = (M+1)(C+O) = (M+1)(C+O)$$

So,

$$P_{opt} = \frac{(M+1)(C+O)}{n}$$

So, based on the year from October 1, 2006 through September 30, 2007, for the state of Virginia:

Total number of policies in force = 104,507

Total claim payments = $14,342,000.00

Estimated average annual premium for Virginia in 2007,

P_{est} = ($14,342,000 payments/104,507 policies in force) × (1.02 cost plus margin) = $164.68

By inspection, we see that the greater n (number of flood insurance policy holders), the lower the premium, because operational costs tend to be relatively fixed.

Table B.45 in appendix B (on page 113) gives the goodness-of-fit and distribution parameters for the state of Virginia. For the premium calculation, the estimated value for C will be different depending on whether you choose to fit the Weibull distribution which failed to reject or the Gamma distribution which was rejected for the State of Virginia using the Kolmogorov-Smirnov (K-S) statistic. By illustrating why it is important to obtain the correct probability distribution, I used the mean of the distributions in calculating the actuarially fair premium or expected cost, C, for Virginia.

Table 1.1: Table showing why it is important to obtain the correct probability distribution.

Parameters	Weibull Distribution	Gamma Distribution	Difference in Mean
alpha	0.3013	0.1760	
beta	27.5500	1384.4000	
Mean	27.55*Gamma Func (4.3185)	0.1760*1384.4	
AFR	250.0930	243.6406	2.65%

So the premium calculations will naturally be different by about 2.6%, depending on the distribution chosen as shown in Table 1.1. The error results from fitting the wrong distribution.

1.3 Literature Review

Risk Management and Insurance

Dillon (2009) define terrorism risk management as a systematic, analytical process to determine the likelihood that a threat will harm individuals or physical assets and to identify actions to reduce risk and mitigate the consequences of a terrorist attack. The definition can easily be extended to all perils.

The insurance industry revolves around managing the losses arising out of the exposures to different *pure* risks. Pure risks are those that could lead to only losses, with no possibility of a gain. Thus insurance coverage does not extend to cover risks associated with investments in the stock market. This Hossack *et al* (1983) explain that the risk theory has been a useful guide to the relationship between reserves, retentions and the level of risk, and the general order of magnitude of these quantities.

The generic risk management principles divide the aggregate losses into two parts, frequency and severity. The discussion of this traditional risk management matrix follows the description of Baranoff (2008).

Table 1.2: This table shows a generic risk management strategy based upon frequency and severity of losses

Traditional Risk Management Matrix

	Low Frequency of Losses	High Frequency of Losses
Low Severity of Losses	Retention—Self-insurance	Retention with Loss control—Risk reduction
High Severity of Losses	Transfer—Insurance	Avoidance

When there is high severity of losses and high frequency of losses, insurance companies are likely to avoid the situation as shown in Table 1.2. This is because there is a high chance of accumulating big losses and debt. Of course, one cannot always avoid risk and not all avoidance necessarily results in zero loss. This is because the avoidance of one peril may create another. For example, a group of people may decide to travel in a car instead of an airplane because of the fear of flying. As these people avoided the possibility of being in an airplane crash, they have on the other hand increased their risk of being in a car accident. Per miles traveled, deaths resulting from car accidents are far greater than that of aircraft victims and thus, the group has increased their probability of injury.

From Table 1.2 low frequency and low severity gives room for risk retention. Here, individual entities self-insure the risk. No matter what the financial loss will be, they will take care of the loss themselves without external insurance company playing any role.

To efficiently retain risk, it is important for insurance companies to make good predictions on losses and its subsequent arrangements made for payment of losses.

When there is high frequency and low severity of losses under the risk management matrix we find control of the frequency of losses to be an effective risk management strategy. Where frequency is largely recorded, steps to avert losses may be useful. This is because individuals and organizations can pay out of their own funds when losses are of low value.

Under the central strategy, efforts are made to lessen the probability of a loss occurring. For example, if one still wants to drive regardless of whether there is snow or sleet, one would want to take instructions to improve his skills of driving to decrease the likelihood of being injured in an accident.

The aim of preventing and reducing losses to the bare minimum involves human activity and expense. At any given time, economic hardships place limits on what may be done, although what is considered too costly at one time may be inexpensive at a later date. To exemplify, in the past, efforts were not made to prevent workplace injuries because the employees were regarded as negligent.

The final element of the risk management matrix involves low frequency and high severity of losses as has been indicated in Table 1.2. This gives room for insurance providers to operate. When there is a low probability of an event occurring coupled with a high severity of losses, this may be successfully managed by transferring risk to an outside party through the purchase of an insurance contract.

An example might be a loss resulting due to danger of the manufacture of a faulty product or an interruption of business due to the damage in a factory. In this case transferring of risk would mean paying someone to take care of some or all of the risk of certain financial losses that cannot be avoided or handled. But Panger and Willmott (1992) explain that prudent decision makers reduce their demand for insurance when excluded losses increase in size or riskiness, absorbing the risk themselves in a calculated way. On the other hand some business risks can also be transferred to their shareholders through the formation of a corporation with limited liability.

In the case of corporations, the owners are faced with the responsibility of paying all debts and other financial obligations when they are faced with a serious possible loss especially when the liabilities of the firm exceed its assets. If the firm is managed by a sole proprietor, he faces the risk by himself. In the case of a partnership of the firm, every partner is responsible without limit for the debts of the firm. In a limited liability firm, they prefer them to be limited to the investment in the corporation without affecting the personal property of shareholders.

Because both individuals and corporations want to transfer risk, it has given room to Risk Pooling where the third party (insurer) brings all the risk exposures together to compute possible future losses with some level of prediction. This leads to a risk transfer where risk is shifted from a person or entity (insured) to a third party.

The insurance contract is a contingent contract, which implies that a cash outflow occurs from the insurance company only when there is a loss to the covered party. The insurance company collects the premiums with a promise to pay for the loss when it occurs in the future. This implies that the insurance company must be able to reasonably predict future losses in order to determine the premiums today. To be able to manage the insurance business, there is the need to have a forecast of events that are likely to happen and how often each event is likely to occur. This leads to a role for probability and statistics in the field of insurance. Thus, a probability distribution of loss arises when there are representations of all possible loss events along with their associated probabilities.

Before insurance companies can manage their risks efficiently, they first need to know the pattern of their losses. Consequently, they collect huge amounts of data (there are insurance pools from which data could be bought for commonly occurring risks, like automobile accidents), and apply statistical analysis to that data. In the following chapter, I focus on the probability distributions that are generally applied in the field of insurance.

Chapter 2: Applications of Statistical Distributions in Insurance

We will now study the application of statistics in general insurance. These statistical methods will aid in assessing premium rates, the amount of risk retained by insurance companies, claims that are outstanding and claims that have been filed. It should be highlighted once again that the successful operations by the insurance industry has been a result of these kinds of statistical data.

In insurance the frequency and severity analysis is of paramount importance because it helps in pricing and product development. Frequency of claims is the number of claims filed per year (period) by policy holders whilst severity of claims is the dollar amount of claims on a per claim basis filed in the given year (period). Cizek *et al* (2005) explains that a typical model for insurance risk has two main components: one characterizing the frequency (or incidence) of events and another describing the severity (or size or amount) of loss resulting from the occurrence of an event. The unexpected increase in the severity and frequency of general insurance claims over the last decade has made the development of useful models for the claims process even more important. Property damage claim frequency, which is the number of property damage claims per 100 insured vehicles, decreased 11 percent whilst property damage claims severity increased 18 percent. Similarly, Baranoff (2008) observed that bodily injury claim frequency decreased 19 percent whilst bodily injury claim severity increased 22 percent during this period of time.

A way of improving usefulness of risk management as a result of the occurrence of an event and its associated probabilities involves the study of statistical distributions.

Both frequency and severity can be studied using statistical distributions. The insurance industry is based on the principle of pooling. Insurance process involves a combination of risk pooling and risk transfer (from the owner of the risk to a third, non-related party) which reduces risks physically and monetarily (Baranoff, 2008). We regard insurance as a social device in which a group of individuals called insureds transfer risk to another party called an insurer in such a way that the insurer combines or pools all the risk exposures together. Pooling the exposures together permits more accurate statistical prediction of future losses (Baranoff, 2008). Pooling reduces the risk because if accurate estimates of the probability distributions are to be made prior to actually providing insurance, then a large number of cases must be considered. If statistical methods are used to determine, for example, the probability of death at age twenty-five, a large number of cases must be observed in order to come up with a reliable estimate.

In this chapter the Normal, Log-normal, Pareto, Gamma, standard Beta, Weibull, Frechet, Gumbel Poisson, binomial and negative binomial are examined. The importance of these distributions in general insurance work is also emphasized.

We will now look at some of the statistical distributions that are used to model the severity and frequency of general insurance claims. The Normal, Log-normal, Pareto and Gamma, Standard Beta (2- parameter) Weibull, Frechet and Gumbel are continuous distributions used to model the severity of claims. On the other hand Poisson, Binomial and Negative binomial are discrete distributions used to model the frequency of claims. Finally I will also discuss some extreme value distributions. There are a host of other distributions that are employed as well, but the ones named above are the most common.

2.1 Continuous Distributions

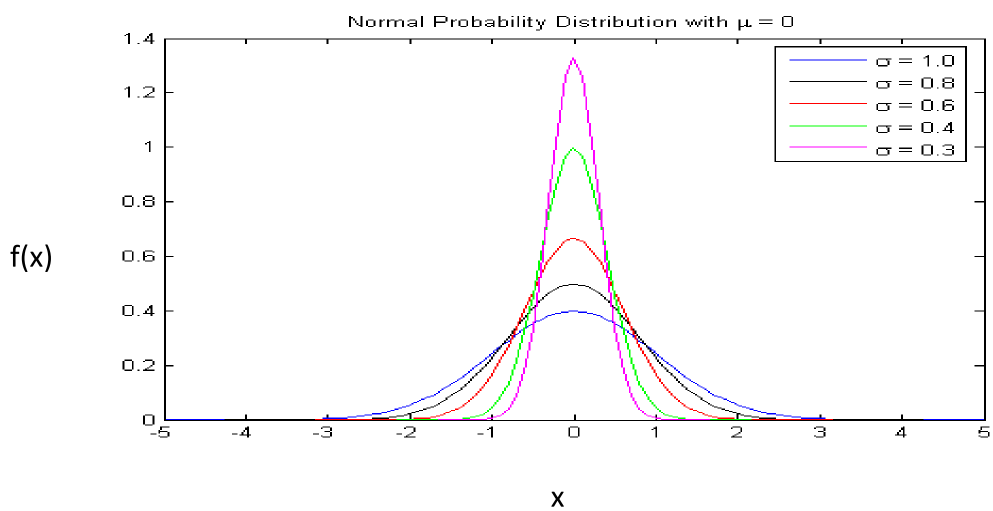
2.1.1 Normal Distribution

The normal distribution is continuous and has 2 parameters, μ and σ. They determine the location and scale, respectively. The importance of the normal distribution in the statistical analysis of insurance is paramount. The probability density function is bell-shaped and symmetrical about the mean. Its formula is given by:

$$f(x\mid\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-(x-\mu)^2/(2\sigma^2)},\quad -\infty<x<\infty,\quad -\infty<\mu<\infty,\sigma>0. \qquad \text{(Casella } et\ al \text{ 2002)}$$

Its mean and variance are given by $EX=E(\mu+\sigma Z)=\mu+\sigma EZ=\mu$ and $VarX=\sigma^2$ respectively, where *x* represent losses or claims.

Figure 2.1. The probability-density function of the normal distribution with μ =0 and different σ values.



Although the range of a normal random variable *x* is from -∞ to ∞, the probability that *x* takes very small or very large values is small. The probability that a normal random variable *X* with mean μ and standard deviation σ lies between two values *a* and *b* is

$$P(a < X < b) = \int_a^b \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left\{-\frac{1}{2}\left[\frac{(x-\mu)}{\sigma}\right]^2\right\} dx.$$

It is not possible to express this integral (area) in terms of explicit mathematical functions. The random variable X is therefore transformed into a variable Z having a mean of 0 and standard deviation of 1. The transformation is

$$Z = \frac{X-\mu}{\sigma}$$

When this transformation is made, X is said to be standardized. It can be shown that if X is normally distributed Z is also normally distributed.

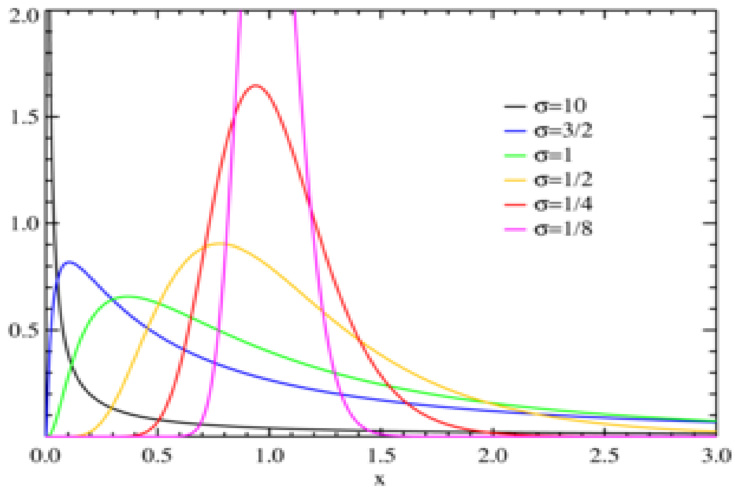
2.1.2 Log-normal Distribution

The second distribution to consider for severity of losses is the log-normal distribution. It is normally used to determine the claim size distribution as it is positively skewed and the random variable does not take negative values, which is a feature of claim size distribution. The lognormal is skewed to the right, and is often useful in modeling claim size (Boland 2007). It has 2 parameters, mean μ which is the location parameter and standard deviation σ the scale parameter. A random variable X is said to have the log-normal distribution with parameters μ and σ if $Y = \ln X$ has the normal distribution with mean μ and standard deviation σ (Hossack *et al 1983*). The probability density function of the log-normal distribution is given by

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[\frac{-1}{2}\frac{(\log x-\mu)^2}{\sigma^2}\right] \; 0 \le x < \infty, -\infty < \mu < \infty, \sigma > 0.$$ (Casella *et al* 2002)

The mean and variance is given by $EX = e^{\mu + (\frac{\sigma^2}{2})}$ and $VarX = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$. Again, x stands for loss or claim amount.

Figure 2.2. The log-normal probability-density function with μ =0 and different σ values.



2.1.3 Pareto Distribution

Pareto distribution is positively skewed, heavy –tailed distribution which is used to model the severity of claims. It has two parameters, *α*, which is the shape parameter and *β*, the scale parameter. For the mean and variance to exist in Pareto distribution β must be greater than 1 and 2 respectively. The random variable X is Pareto with (positive) parameters α and β if it has density function

$$f(x\,|\,\alpha,\beta)=\frac{\beta\alpha^{\beta}}{x^{\beta+1}}, \quad a<x<\infty\,,\ \alpha>0\,,\ \beta>0\,. \qquad \text{(Casella } et\ al \text{ 2002)}$$

The Pareto distribution is named after Vilfredo Pareto (1848-1923) who used it in modeling welfare economics. Today, it is commonly used to model income distribution in economics or claim-size distribution in insurance, due in large part to its extremely thick tail. Like the exponential family of random variables, the Pareto distributions have density and survival functions which are very tractable. Pareto random variables have some nice preservation properties. For example, if X ~Pareto (α,λ) and $\text{k}>0$, then kX ~ Pareto $(\alpha,\kappa\lambda)$ since

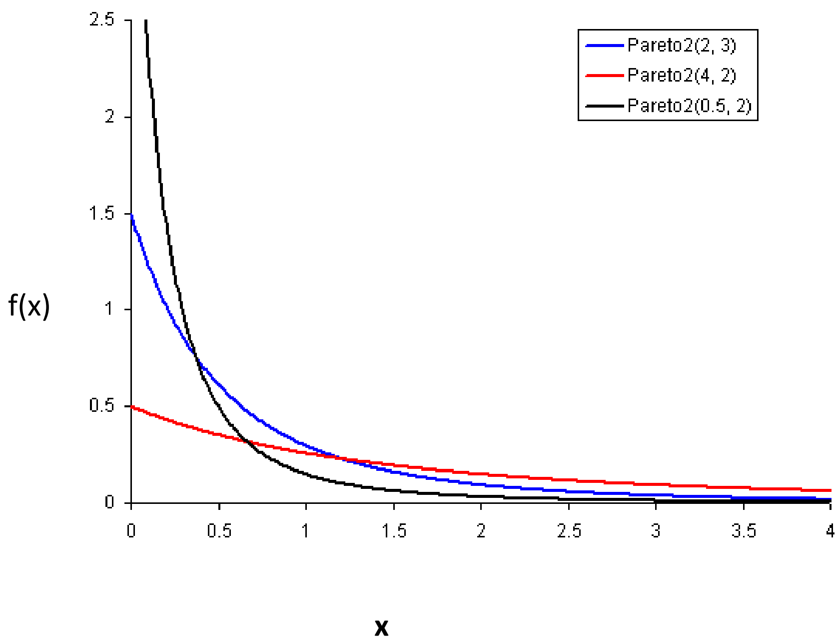
$$P(kX > x) = P(X > x/k) = \left(\frac{\lambda}{\lambda + x/k}\right)^{\alpha} = \left(\frac{k\lambda}{k\lambda + x}\right)^{\alpha}.$$

This property is useful in dealing with inflation in claims.

The mean and variance of the Pareto distribution is given by $EX = \frac{\beta\alpha}{\beta - 1}$, where $\beta > 1$ and

$VarX = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$, where $\beta > 2$.

Figure 2.3. The Pareto probability-density function with different α and β values.



2.1.4 Gamma Distribution

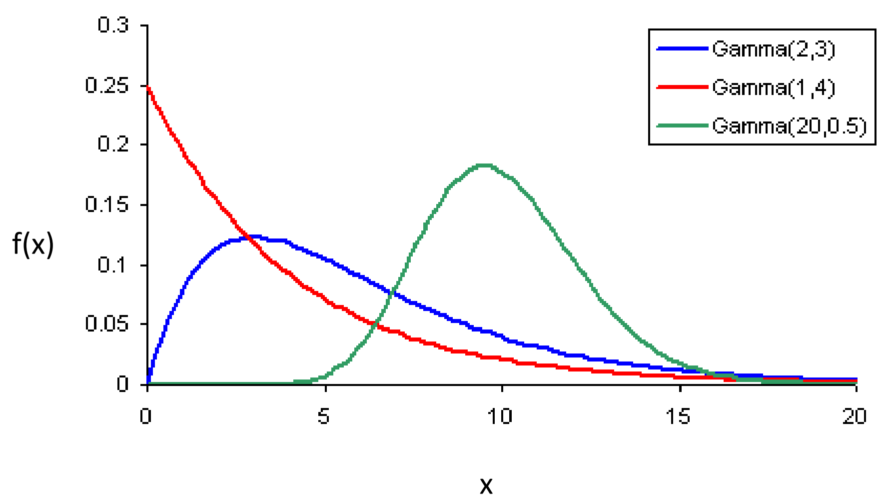
The next distribution to discuss is the gamma distribution which is used in the study of claim size and in the analysis of heterogeneity of risk. It has two parameters, α, which is the shape parameter and β, the scale parameter.

The probability density function for gamma distribution is given by

$$f(x \mid \alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta},\ 0 \le x < \infty\ ,\alpha,\beta > 0. \qquad \text{(Casella } et\ al\ \text{2002)}$$

Γ (α) is a number which depends on *α*. The gamma distribution is not symmetrical; instead it is positively skewed, but as it increases, the skewness decreases and the distribution becomes more symmetrical. The mean and variance are given by $EX = \alpha\beta$ and $VarX = \alpha\beta^2$.

Figure 2.4. Probability density-function of the Gamma distribution with different α and β values.



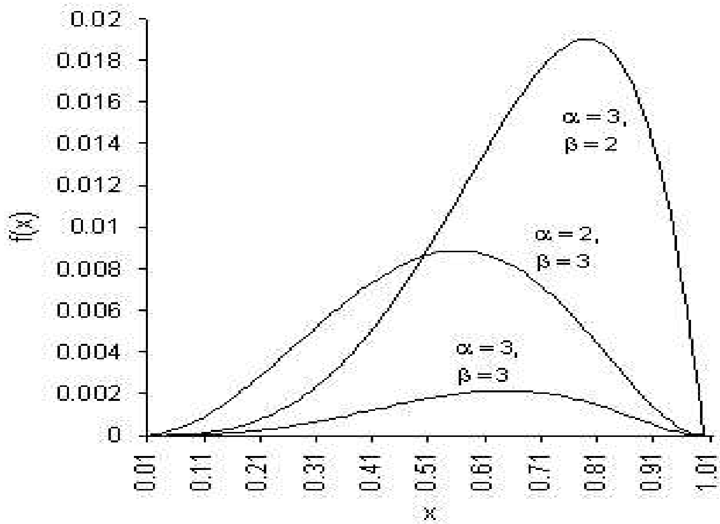
2.1.5 Standard Beta Distribution

The generalized two-parameter beta distribution (standard beta) with parameters α and β is another frequently used distribution for a continuous random variable with interval $0 \le x \le 1$. It is helpful for modeling proportions. Its probability density function is given by

$$f(x \mid \alpha,\beta) = kx^{\alpha-1}(1-x)^{\beta-1} \qquad \text{(a)}$$

for $0 \le x \le 1$, where $\alpha\ ,\ \beta\ >\ 0$.

Figure 2.5. Probability density-function of the standard Beta distribution with different parameters values.



It is important to know that $\alpha, \beta > 0$ determine the shape of the curve, and k is a scalar we need to make this a probability density function. K is given by

$$k = \frac{1}{B(\alpha, \beta)} \qquad \text{(b)}$$

where B is the beta function.

Substituting equation (b) into equation (a) which is the density function will give us

$$f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1} \ldots \text{(c)}$$ (Casella *et al* 2002)

for $0 \le x \le 1, \alpha, \beta > 0$,

where $B(\alpha, \beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1} dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. By substitution the probability density function of the standard beta (α, β) distribution becomes

$$f(x:\alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}.$$

So, $k=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ is the constant we need to make the curve whose shape is given by $x^{\alpha-1}(1-x)^{\beta-1}$ a density function. The shape of the beta distribution curve is different depending on the values of α and β which makes the beta (α,β) a family of distributions. The uniform distribution has a relationship with the beta distribution where $\alpha=1$ and $\beta=1$.

The mean and variance of the standard beta distribution is given by $EX=\frac{\alpha}{\alpha+\beta}$ and

$VarX=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

2.1.6 Weibull Distribution

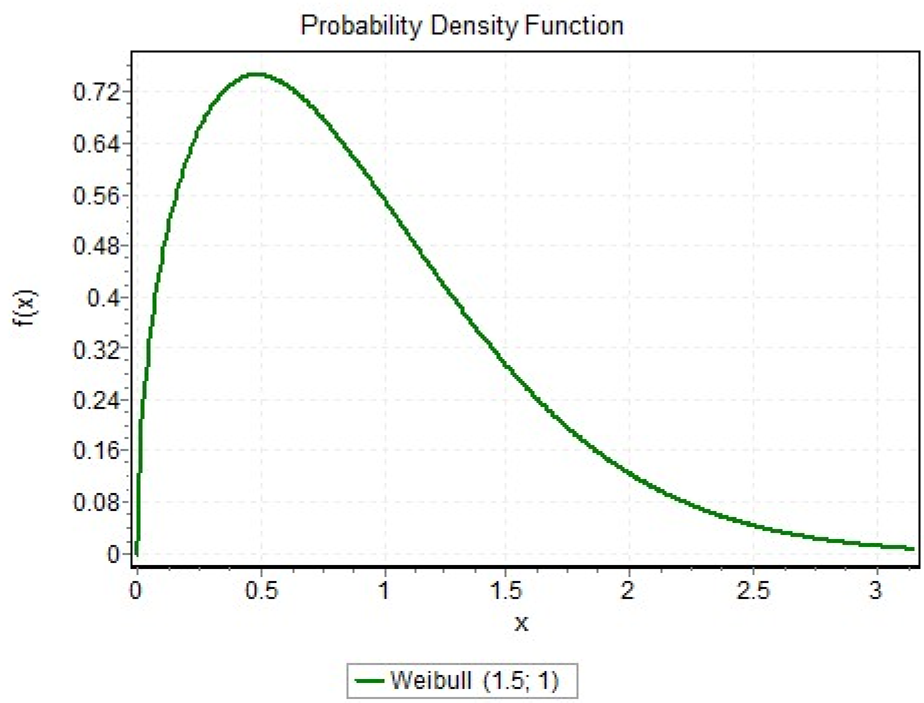
The random variable X with Weibull distribution has a probability density function given by

$$f(x\mid\gamma,\beta)=\frac{\gamma}{\beta}x^{\gamma-1}e^{\frac{-x^{\gamma}}{\beta}}\ ,0\le x<\infty\ ,\ \gamma>0,\ \beta>0.$$ (Casella *et al* 2002)

It has two parameters γ, which is the location parameter and β, the scale parameter. Kleiber and Kotz (2003) explains that the Weibull distribution has no doubt received maximum attention in the statistical and engineering literature of the last ten years and is still going strong. In economics it is probably less prominent, but D'Addario (1974) noticed its potentials for income data and Hogg and Klugman (1983) for insurance losses. A simple argument leading to a Weibull distribution as a distribution of fire loss amount was given by Ramachandran (1974).

A fit to a small data set (35 observations) of hurricane losses was carried out using the Weibull distribution by Hogg and Klugman (1983) in the actuarial literature and the authors found that it performs about as well as the lognormal distribution. In the study employing 16 loss distributions in the Cummins *et al*. (1990), the Weibull distribution does not provide an adequate fit to the Cummins and Freifelder (1978) fire loss data. In particular the data appear to involve a model with heavier tails such as an inverse Weibull distribution. However, in practice, it is often found that the Weibull distribution frequently does significantly better than the more popular lognormal distribution.

Figure 2.6. The probability density-function of the Weibull distribution with α =1.5 and β =1.



Its mean and variance are given by $EX = \beta^{1/\gamma}\Gamma(1+\frac{1}{\gamma})$ and $VarX = \beta^{2/\gamma}\left[\Gamma(1+\frac{2}{\gamma}) - \Gamma^2(1+\frac{1}{\gamma})\right]$.

2.2 Discrete Distributions

We will now turn and look at some of the discrete distributions that are used to model the frequency of claims. These distributions include Poisson, Binomial, and Negative Binomial.

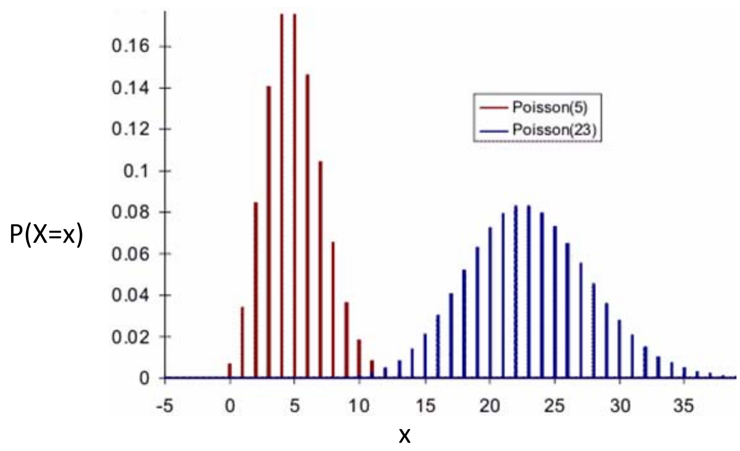
2.2.1 Poisson Distribution

The Poisson distribution is commonly employed for analyzing the incidence of claims and is also a non-negative, integer-valued distribution which plays an important role in statistical theory. The Poisson distribution which is a generally applied discrete distribution can be used as a model for a number of diverse types of experiments. If we model an event in which we are waiting for an occurrence such as waiting for a bus, then the number of occurrences in a given time interval can at times be modeled by the Poisson distribution. The Poison distribution was built on one of the theories that, for small time intervals, the probability of an arrival is proportional to the length of waiting time. It is therefore meaningful to think that the longer we wait, the more likely it is that a customer will board the bus. A random variable *X*, taking values in the nonnegative integers, has a Poisson (λ) distribution if

$$P(X = x/|\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{, where x = 0, 1, 2…; } 0 \le \lambda < \infty \qquad \text{(Casella \textit{et al} 2002)}$$

The parameter λ is both the mean and the variance of the distribution where λ is a positive real number which is equal to the expected number of occurrences that occur during the given interval and x the number of claims.

Figure 2.7. The probability-mass function of the Poisson distribution with means 5 and 23 respectively.



Its mean and variance are given by $EX = \lambda$ and $VarX = \lambda$.

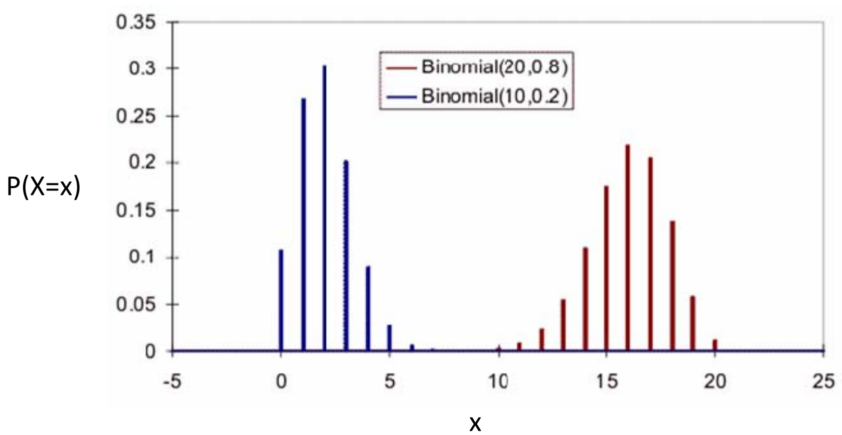
2.2.2 Binomial Distribution

The binomial distribution is also useful for analyzing claim frequencies and has n and p as its parameters, where n (positive integer) is the number of trials and p is the probability of success. A discrete random variable X has a binomial distribution if its probability mass function is of the form

$$P(X = x \mid n, p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad ; \quad x = 0, 1, 2, \ldots, n; \quad 0 \le p \le 1 \qquad \text{(Casella } et\ al \text{ 2002)}$$

The binomial distribution, one of the more useful discrete distributions, is essentially a Bernoulli trial, repeated n times.

Figure 2.8. The probability-mass function of the binomial distribution with n=10, p= 0.2 and n=20, p=0.8 respectively.



The mean and the variance is given by $EX = np$ and $VarX = np(1-p)$.

2.2.3 Negative Binomial Distribution

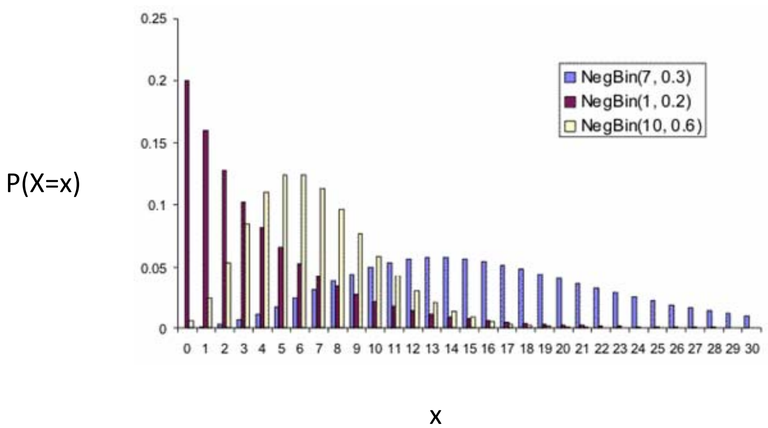
The number of trials up to and including the rth success in a sequence of independent Bernoulli trials with a constant success probability p has negative binomial distribution with parameters p and r. Let the random variable X denote the trial at which the rth success occurs, where r is a fixed integer. The probability mass function

The probability mass function

$$P(X = x \mid r, p) = \binom{r+x-1}{x} p^r (1-p)^x \;; \quad x = 0,1,.....; \;\; 0 \le p \le 1 \qquad \text{(Casella } et\ al \text{ 2002)}$$

And we say that X has a negative binomial (r, p) distribution with an expected value and variance $EX = \frac{r(1-p)}{p}$ and $VarX = \frac{r(1-p)}{p^2}$. Again x refers to the frequency of losses.

Figure 2.9. The negative binomial probability-mass function with different *r* and *p* values.



The most important application of the negative binomial distribution, as far as general insurance applications are concerned, is in connection with the distribution of claim frequencies when the risks are not homogenous.

2.3 Extreme Value Distributions (EVD)

Extreme value distribution is a family of continuous probability distributions which is developed within extreme value theory to combine the Gumbel, Frechet and Weibull families also known as Types I, II and III extreme value distributions. Extreme value theory plays an increasingly important role in stochastic modeling in insurance and finance. It can be used in applications involving natural phenomena such as rainfall, floods, wind gust, air pollution, corrosion, etc. Extreme value theory deals with the behavior of the maximum and minimum of independent identically distributed random variables whereby their properties are determined by the upper and lower tails of the underlying distribution.

Fisher and Tippett (1928) published results of an independent inquiry into extreme value distributions. Since 1920's there have been a number of papers dealing with practical applications of extreme value statistics in distributions of human lifetimes [Gumbel (1937)], strength of materials [Weibull (1939)], flood analysis [Gumbel (1941, 1944, 1945)] to mention a few examples. With regards to application, Gumbel made several important contributions to the extreme value analysis. Gumbel was the first to call the attention of engineers and statisticians to possible applications of the formal extreme value theory to certain distributions which had previously been treated otherwise. Another important early publication related to extreme value analysis of the distribution of feasible strengths of rubbers is due to S. Kase (1953). There are several books that deal with extreme value distributions and their statistical applications. Castillo (1988) has successfully presented many statistically applications on extreme value theory with emphasis on engineering problems. Beirlant, Teugels and Vynekier (1996) provided a clear practical analysis of extreme values with emphasis on actuarial applications.

2.3.1 Types of Extreme Value Distributions

There are three types of extreme value distributions namely:

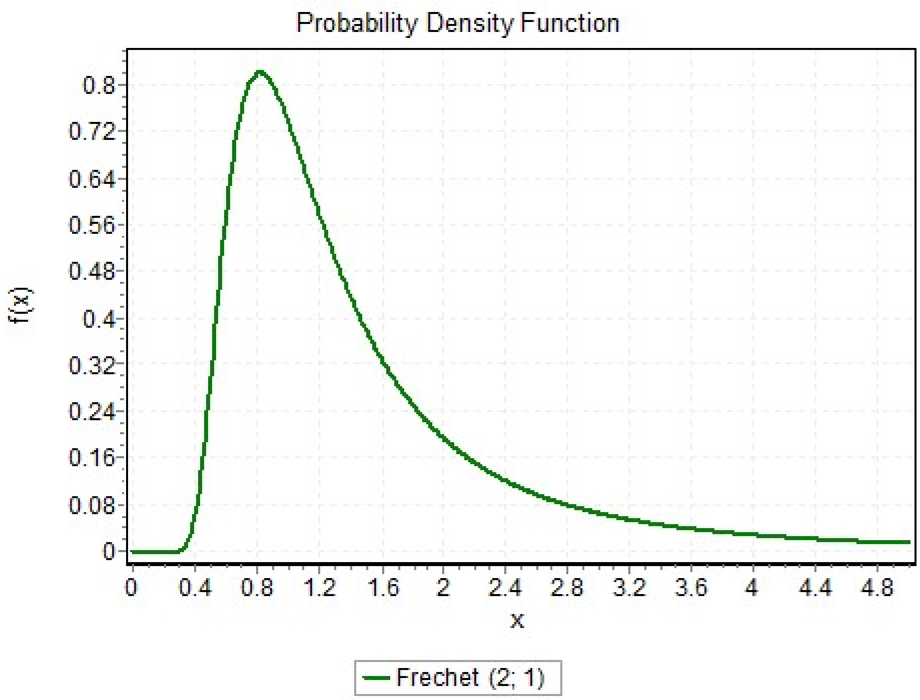
(1) Gumbel-type distribution (EVD type I):

$$\Pr[X \le x] = \exp[-e^{-(x-\mu)/\sigma}]. \tag{1.1}$$

(2) Frechet-type distribution (EVD type II):

$$\Pr[X \le x] = \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\xi}\right\} \ , x \ge \mu. \tag{1.2}$$

Figure 2.10. The probability-density function of the Frechet distribution with μ=2 and σ=1.



(3) Weibull-type distribution (EVD type III): Refer to Figure 2.5 for graph

$$\Pr[X \le x] = \exp\left\{-(\frac{\mu - x}{\sigma})^{\xi}\right\} \quad ,x \le \mu. \qquad (1.3)$$

where μ , $\sigma(>0)$ and $\xi(>0)$ are parameters. It can be observed that Frechet and Weibull distributions are related by a simple change of sign. Type II and III distributions can be transformed to type I distributions by the simple transformations

$$Z = \log(X - \mu),\ Z = -\log(\mu - X),$$

respectively. The reason why extreme value is attached to these distributions is because they can be obtained as limiting distributions $(n \to \infty)$ of the greatest value among n independent random variables each having the same continuous distribution.

The three types of distributions in (1.1)-(1.3) may all be represented as members of a single family of generalized distributions with cumulative distribution function

$$\Pr[X \le x] = [1+\xi(\frac{x-\mu}{\sigma})]^{-1/\xi},\ 1+\xi(\frac{x-\mu}{\sigma}) > 0,\ -\infty < \xi < \infty,\ \ \sigma > 0. \tag{1.4}$$

The generalized extreme value distribution (GEV) has three parameters (Reiss and Thomas, 2000): the location parameter μ, the scale parameter σ, and the shape parameter ξ, which reflects the fatness of tails of the distribution (the higher value of this parameter, the fatter tails). When $\xi > 0$, equation (1.4) is equal to that of (1.2). When $\xi < 0$, equation (1.4) is equivalent to that of (1.3). Lastly, when $\xi \to \infty$ or $-\infty$, equation (1.4) turn out to be the type 1 extreme value distribution in (1.1). That is why the distribution function in (1.4) is referred to as the generalized extreme value distribution and is also at times called the von Mises type extreme value distribution or the von Mises-Jenkinson type distribution.

Chapter 3: Estimation

3.1 Maximum Likelihood Estimate

One of the procedures for calculating a point estimator of a parameter is through the method of maximum likelihood, which was developed by a famous British statistician Sir R.A. Fisher in 1920. Salkind (2007) states that maximum likelihood estimate (MLE) of a parameter is the value that gives the observed data the highest probability possible.

We will consider X to be a random variable with probability distribution $f(x:\theta)$, where θ is a single unknown parameter. If we let $x_1, x_2, ..., x_n$ be the observed values in a random sample of size n, then the likelihood function of the sample will be given by

$$\text{L}(\theta) = f(x_1;\theta).f(x_2;\theta)...f(x_n;\theta).$$

This means that the likelihood function now becomes a function of the unknown parameter, which is θ in this case. The ML estimator of θ is the value of θ that maximizes the likelihood function $L(\theta)$ (Montgomery and Runger 2003). When x is discrete the likelihood function of the sample $L(\theta)$ will be the probability

$$P(X_1 = x_1, X_2 = x_2, ..., X_n = x_n)$$

which will also mean that $L(\theta)$ will be the probability of getting the values of the sample $x_1, x_2, ..., x_n$.

In the case where the random variable is discrete, the maximum likelihood estimator is one that makes best use of the probability of occurrence of the sample values. That makes the interpretation of the likelihood function clear in the scenario where the random variable is discrete. The method of maximum likelihood provides estimators which are usually quite satisfactory (Hossack *et al* 1983). MLE can be biased depending on the sample size.

Where the sample size is small, the bias of the maximum likelihood estimators can be regarded as significant. The principle of MLE possesses certain characteristics when the sample size n is large and if $\hat{\theta}$ is the estimator of the parameter θ. These characteristics include:

(1) The ML estimator θ should be unbiased, so that its expectation is equal to the true value of the parameter. Thus, the estimate obtained should be equal to the underlying parameter and should not provide estimates which are too high or too low.

(2) The ML estimator $\hat{\theta}$ has an approximate normal distribution.

(3) The MLE is asymptotically efficient for large samples under quite general conditions. That is, the variance of the estimator should be minimal.

Please note that the distributions fitted in this thesis for the flood loss data are fitted using the maximum likelihood estimation technique.

The maximum likelihood estimator of the Poisson claim frequency rate is merely the mean number of claims per policy per annum.

Though the method of maximum likelihood is the most frequently used, there are other methods of obtaining estimators such as the method of moments and least squares. Out of the three standard methods, the method of moments is perhaps the most readily understood and easiest to compute. Both the method of maximum likelihood and the method of moments can produce unbiased point estimators.

3.2 Method of Moments

The main idea behind the method of moments is to equate the population moments which are given in terms of expected values, to the corresponding sample moments. For example the point estimate of the mean from the method of moments is found by setting the sample mean equal to the population mean and so on.

Suppose $X_1, X_2, ..., X_n$ is a random sample from the probability distribution *f(x)*. In this case *f(x)* can either be a discrete probability mass function or a continuous probability density function. The K^{th} population moment is given by $E(X^k)$ where k =1,2,**...**. The corresponding K^{th} sample moment is $(\frac{1}{n})\sum_{i=1}^{n} X_i^k$ where k =1, 2.**...** To show this technique, the first population moment is $E(X) = \mu$**,** and the first sample moment is $(\frac{1}{n})\sum_{i=1}^{n} X_i = \bar{X}$**.** If we equate the population and the sample moments, $\hat{\mu} = \bar{X}$. This makes the sample mean become the moment estimator of the population mean. In general, the population moments will be a function of the unknown parameters of the distribution, say $\theta_1, \theta_2, ...\theta_m$.

Let $X_1, X_2, ..., X_n$ be a random sample from either a probability mass function or a probability density function where $\theta_1, \theta_2, ...\theta_m$ are m unknown parameters. To find the moment estimators $\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_m$, we will equate the first m population moments to the first m sample moments and solve the resulting equations for the unknown parameters. Hossack *et al* (1983) explains that in the case of a two-parameter distribution, for example, we compute the first two moments of the sample and equate these to the corresponding theoretical moments of the distribution. We will consider an example of this technique.

Example: Table 3.1 summarizes the claim sizes of a sample of 100 claims on an insurance company. Assuming that the log-normal is a suitable model, I will obtain estimates of its parameters, μ and σ, and estimate the probability that a particular claim exceeds $4000.

Table 3.1: Claim size distribution

Claim size ($)	Number of Claims
0-400	2
400-800	24
800-1200	32
1200-1600	21
1600-2000	10
2000-2400	6
2400-2800	3
2800-3200	1
3200-3600	1
Over 3600	0
Total	100

Assuming that the number of claims of Table 3.1 refer to claims with sizes equal to the mid-point of the respective claim size interval, we obtain the mean claim size of the observed distribution as follows:

$$\begin{aligned}\text{Mean claim size} = &\ \$(200\times\frac{2}{100}+600\times\frac{24}{100}+\ldots+3400\times\frac{1}{100})\\ &= \$1216\end{aligned}$$

The variance of the observed claim size distribution is calculated as follows:

$$\begin{aligned}\text{Variance} = &\ (200^2\times\frac{2}{100}+600^2\times\frac{24}{100}+\ldots+3400^2\times\frac{1}{100})-1216^2\\ &= 362,944\end{aligned}$$

The mean and variance of the log-normal distribution are given by

$Mean = \exp(\mu + \frac{1}{2}\sigma^2);$ and $Variance = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1]$.

To estimate μ and σ^2 we therefore equate the above mean and variance to the observed values 1216 and 362,944 respectively. Thus,

$\exp(\mu + \frac{1}{2}\sigma^2) = 1216$ and $\exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1] = 362,944$

Squaring the first of these equations and dividing the second equation by this square we obtain

$$\exp(\sigma^2) - 1 = 0.2455,$$

From which

$$\sigma = 0.469,$$

and so

$$\mu = 6.993$$

The probability that a particular claim X exceeds \$4000 is equal to the probability that lnX exceeds 8.294. But $\ln X$ is normally distributed with mean μ and standard deviation σ, and we have estimated μ to be 6.993 and σ to be 0.469. An estimate of the required probability is, therefore,

$$1 - \Phi(\frac{8.294 - 6.993}{0.469}) = 1 - \Phi(2.77) = 0.00280$$

In other words, we estimate that about 3 claims in 1000 will exceed \$4000.

While the above examples illustrate the application of statistical distributions in the area of risk and insurance, it is another statistical task to determine whether a given distribution fits the given insurance loss data or not. We will tackle this question in the next chapter.

Chapter 4: Fitting Loss Distribution Using Different Test Statistics

It can be both exciting and a demanding exercise to fit a probability distribution to claim data. Boland (2007) explains that when one is trying to fit a distribution to claim data, it is well worth remembering the famous quote of George Box who states that all models are wrong, some models are useful. In the preceding section we have talked about the method of maximum likelihood (ML), and the method of moments (MM) in estimating parameters of typical loss distributions. But, how do we make a decision on the particular type of distribution, the method of estimation, and ensure the resulting distribution provides a good fit? Exploratory Data Analysis (EDA) techniques such as histograms, qq plots and box-plots can often be useful in investigating the suitability of certain families of distribution (Boland 2007).

Because these techniques in examining the fit is tentative, one would have to make use of one or more of the usual typical methods to test fitness such as the Kolmogorov – Smirnoff (K-S), Anderson – Darling (A – D), or chi-square goodness of fit tests. The K – S and A – D tests are used to test continuous distributions, while the chi – square goodness of fit test is used to test both continuous and discrete distributions (Boland 2007).

4.1 Kolmogorov – Smirnov test

The Kolmogorov-Smirnov statistic is a method used to test if there is any difference between the cumulative distribution function of the sample data and the cumulative probability distribution function. The test is based on the maximum absolute difference between the cumulative distribution functions of the samples from each population.

The Kolmogorov- Smirnov (K-S) test helps test the null hypothesis H_0 that a sample x comes from a probability distribution with cumulative distribution function (cdf) F_0. The K – S two-sided test rejects the hypothesis H_0 if the maximum absolute difference d_n between F_0 and the empirical cumulative distribution function (ecdf) $\hat{F}_n$ is large. The K – S test statistic is given by:

$$d_n = \sup_{-\infty<x<\infty} \left| \hat{F}_n(x) - F_0(x) \right| \qquad \text{(Boland 2007)}$$

F_n is defined as

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{X_i \le x}$$

for n independent and identically distributed observations X_i, where $I_{X_i \le x}$ is the indicator function, equal to 1 if $X_i \le x$ and equal to 0 otherwise. The K – S statistic is nonparametric and the null distribution of d_n is the same for all continuous distribution functions F_0. This makes it possible using one set of critical values as far as this test statistic is concerned. However, a test has its flaws, and in particular, is frequently not good in detecting tail discrepancies and sometimes the upper tail of a loss distribution is usually of considerable interest. The K – S is constant under transformations where one can test that a data set x comes from a distribution with cdf F_0 or whether the transformed sample data $F_0(x)$ comes from, say, another distribution.

4.2 Anderson-Darling test

The Anderson-Darling (A-D) test is also used to test whether a given set of data fits a specified probability distribution. Apart from using the test to see if a data fits the distribution, it can also be used in estimating parameters using the minimum distance estimation approach. The

A-D test is an adjustment of the Kolmogorov-Smirnoff test which takes into consideration the tails of the distribution. Because of the sensitive nature of the test, it has the disadvantage that it is not a nonparametric test, and before you can get the critical values for the test statistic, calculations will have to be made for each distribution being considered. There are many software packages that tabulate critical values for the A-D test statistic when we want to find out if a particular distribution such as normal, lognormal, gamma or weibull fits the data. The A-D test statistic A_n^2 for a sample x of size n from the null distribution function F_0 and the corresponding density function f_0 is given by

$$A_n^2 = n\int_{-\infty}^{+\infty} \frac{[F_0(x) - \hat{F}_n(x)]^2}{F_0(x)[1 - F_0(x)]} f_0(x)dx.$$

(Boland 2007)

$\hat{F}_n(x)$ is a step function with jumps at the order statistics $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$, and also for computational purposes the following expression will be useful:

$$A_n^2 = -\sum_{i=1}^{n} \frac{2i-1}{n}\{\log[F_0(x_{(i)})] + \log[1 - F_0(x_{(n+1-i)})]\} - n$$

4.3 Chi-square goodness-of-fit tests

The chi-square goodness-of-fit test is mostly used to test how well a particular distribution fits a given data set, be it discrete or continuous. The test has an assumption of being asymptotic where the test of fit for a specified distribution is basically condensed to a multinomial setting. Boland (2007) explains that when testing the fit of a continuous distribution, the data is usually first binned (or grouped) into k intervals of the form $I_i = [c_i, c_{i+1})$, for $i = 1, \ldots, k$, although this clearly involves losing information in the sample. We then calculate the number of expected observations E_i based on a grouped data and compare it with the actual observed numbers O_i for each interval. We then measure the fit of the hypothesized null distribution which is obtained from the test statistic:

$$\chi^2 = \sum_{1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

This also compares observed and expected values. If the value of the chi-square test statistic is large, we will reject the null hypothesis being considered since it signifies a lack of fit between the observed and expected values. We would reject the hypothesis that the distribution of the population is the hypothesized distribution if the calculated value of the test statistic $\chi^2 > \chi^2_{\alpha,k-p-1}$ (Montgomery *et al* 2003). The null hypothesis usually is the population that follows the hypothesized distribution and χ^2 has, approximately, a chi-square distribution with *k*-*p*-1 degrees of freedom, where *p* is equal to the number of parameters of the hypothesized distribution which is calculated by sample statistics. If we have to estimate the parameters of a grouped or ungrouped data, then the number of degrees of freedom will largely depend on the method of estimation.

Chapter 5: Distribution Fitting to Flood Losses – Related Aspects

5.1 How to Measure Spatial Association and Correlation

Geographic information system (GIS) has brought about new method of exploratory data analysis that center on spatial data. A GIS is a collection of computer software tools that facilitate, through georeferencing, the integration of spatial, nonspatial, qualitative, and quantitative data into a database that can be managed under one system environment (Burrough 1986). Spatial data also known as geospatial data or geographic information is the data or information that identifies the geographic location of features and boundaries on Earth, such as natural or constructed features, oceans and more. This is why it is important to have a more formal way of assessing whether observations are spatially clustered or interrelated across some forms of ties between observations. This Cressie (1993) explains that data close together in time or space are likely to be correlated and has been used successfully by statisticians to model physical and social phenomena. In the case of this study, discovering such associations, however, will mean that we have some idea about which States are likely to be connected to one another.

Nonetheless for many purposes we try to find how observations are connected. Graphs and matrices are used to represent connectivities between observations. For example if we define a binary matrix C that specifies how individual observations are connected, then we will have an entry $C_{ij} = 1$ if two observations i and j are regarded as connected. If they are not, then $C_{ij} = 0$.

Spatial association for example, in the case of measures of democracy would mean how close countries were to one another in terms of their spatial measurement and whether there was a correlation existing among these countries. Gleditsch and Ward (2008) explain that the first task in formally assessing such correlations is to specify the interdependencies among data. This will call for developing a list of which observations are connected to one another. Another way to establish connectivity is by the physical distance, say the distance between States of the United States of America and also specifying that States are neighbors if they are adjacent to one another. Gleditsch and ward (2008) developed a database of the minimum distances among all countries in the world.

We will now look at tables showing the adjacency matrix format and the connections among States.

Table 5.1: Adjacent and Non-Adjacent Matrix Format

	Alabama	Delaware	Florida	Georgia	Maryland	S.Carolina	Tennessee	Virginia
Alabama	0	0	1	1	0	0	1	0
Delaware	0	0	0	0	1	0	0	0
Florida	1	0	0	1	0	0	0	0
Georgia	1	0	1	0	0	1	1	0
Maryland	0	1	0	0	0	0	0	1
S. Carolina	0	0	0	1	0	0	0	0
Tennessee	1	0	0	1	0	0	0	0
Virginia	0	0	0	0	1	0	1	0

Note: A connection is present if States are adjacent to one another.

Table 5.2: Adjacency Matrix for a Subset of United States of America

States	Adjacency
Alabama	Florida, Georgia, Tennessee
Delaware	Maryland
Florida	Alabama, Georgia
Georgia	Alabama, Florida, South Carolina, Tennessee
Maryland	Delaware, Virginia

South Carolina	Georgia
Tennessee	Georgia
Virginia	Maryland, Tennessee

A subset of the United States is shown in Table 5.1 which demonstrates the corresponding binary matrix C of these connections. This will help us test if the average correlations between losses of adjacent States are different from the average correlations of losses between non-adjacent States. Table 5.2 on the other hand represents a subset of adjacent States as a list in the United States. In dealing with small subsets, it is much easier to derive spatial characteristics and document them as lists of connections. A matrix representation also helps in describing certain variables or measures reflecting spatial structures and variations. One way to do this is to find out whether two connected observations i and j are alike to one another – that is to establish whether high or low values for i tend to go together with high or low values for j. Since i is most of the time linked to many observations, we will not have spatial association unless it looks the same as its neighbors. Once established, we try to find out how these adjacencies should be handled in the analysis itself. The question we try to ask ourselves is whether to give equal weights to adjacent States or weigh some differently according to the measure of their size or importance. To put together information regarding adjacent States, we generally assume that all neighbors have equal weight which is proportional to 1 over the total number of connectivities. This notwithstanding, there might be other weighting schemes by researchers provided it makes sense in the context of their specific research questions. If we consider regression models with a row-normalized matrix, the sum of all adjacent weights add to 1. For normalization to make sense in a specific application, it will depend on the problem under discussion. Murdoch *et al*. (1997), for example, are interested in how a country's emissions of pollutants are influenced by depositions from other countries. What is important is the total amount of emitted pollutants and therefore normalizing the adjacency matrix by the number of adjacent countries is perhaps not the right thing to do.

Testing for spatial dependence and correlation can be done using Moran's I statistic which Moran (1950a &1950b) explains is the linear association between a value and a weighted average of its neighbors, a global correlation of the values of an observation with those of its neighbors. Moran (1950a) proposed a test of spatial dependence between sites. The generalized Moran's I is given by a weighted, scaled cross-product:

$$I = \frac{n\sum_i \sum_{j\neq i} w_{ij}(y_i - \bar{Y})(y_j - \bar{Y})}{(\sum_i \sum_{j\neq i} w_{ij})\sum_i (y_i - \bar{Y})^2},$$

where w denotes the elements of the row standardized weights matrix W and y is the variable of concern. I can be considered normal with mean equal to $-1/(n-1)$. The variance of Moran's I is given by

$$\operatorname{var}(I) = \frac{n^2(n-1)\frac{1}{2}\sum_{i\neq j}(w_{ij}+w_{ji})^2 - n(n-1)\sum_k(\sum_j w_{kj} + \sum_i w_{ik})^2 - 2(\sum_{i\neq j} w_{ij})^2}{(n+1)(n-1)^2(\sum_{i\neq j} w_{ij})^2}$$

If we standardize the variable of concern as z_i, Moran's I is simply

$$I = \frac{1}{2}\sum_{ij} c_{ij} z_i z_j \qquad \forall i \neq j .$$

I undertake a bivariate correlation analysis of flood losses and find that maximum and minimum correlations amongst adjacent States are 0.990 (between Pennsylvania and New York) and -0.123 (between Arizona and Nevada); and 0.986 (between Pennsylvania and New Mexico) and -0.189 (between Florida and Louisiana) for non-adjacent States. Below is the summary statistics on correlation as shown in Table 5.3.

Table 5.3: Summary statistics on correlation

	Mean	Median	Minimum	Maximum	Standard Deviation
Adjacent States	0.215	0.0896	-0.123	0.990	0.288
Non-Adjacent States	0.039	-0.0293	-0.189	0.986	0.177

5.2 Consumer Price Index (CPI) Inflation Calculator

This is an inflation calculator used to adjust the cost from one year to another. The value of the index has been calculated every year since 1913 through to 2009. The inflation calculator depends on the average inflation index during the calendar year. The inflation calculator is able to calculate the rate of inflation from and to any of the range of years as stated above. The relative value in prices of all goods and services purchased for consumption by urban households is termed the CPI.

The consumer price index which is also called an inflation indicator is calculated by the Bureau of Labor Statistics (BLS). The CPI which is published every month is what the United States BLS uses to indicate the rate of inflation. Though very simple in calculation, it serves a very important purpose. The difference in comparing the prices of everyday goods from one month to the other represents the CPI. If the CPI number increases extremely, then this signifies that cost of living is high which will mean there is inflation. Inflation causes the Federal Reserve to take suitable measures to control it and has also helped to decide on the changes that need to be made on interest rates.

For the sake of this study, the CPI calculator is utilized to convert flood losses in norminal terms into flood losses in real terms.

Chapter 6: Fitting a Distribution to Flood Loss Data

6.1 Identifying the distribution that best fits aggregate loss data

In estimating the parameters of a number of possible probability distributions, we try to decide which, if any, gives the best representation of our data and to find out which distribution best fits a particular data. Even though eleven different types of distributions are mentioned, I will use seven to fit the flood loss data of the 50 States of the United States. They are the standard Beta, Frechet, Gamma, Gumbel, Lognormal, Normal, and Weibull distributions. A goodness-of-fit test based on the linearity of the probability plot (Rice 2007) is shown as well as a formal statistical test of distribution fitting.

6.2 Probability Plot

Chambers (1983) describes a probability plot as a graphical technique for assessing whether or not a data set follows a given distribution such as the normal or Weibull. It is a plot of the quantiles (or percentages) of points below a given value of the sample data set against the quantiles of the postulated probability distribution (Lewis 2004). A straight line which is also called the reference is also plotted. If the plotted points fall along the reference line, then it means that the sample comes from the proposed probability distribution. Departures of the sample from the reference line would mean departures from the postulated distribution. If the data points do not lie next to the reference line, then a different probability model will have to be chosen.

6.3 Formal Test Statistics

Various test statistics can be used to assess the fit of a postulated severity of loss probability model to empirical data. Even though three test statistics are mention in this chapter, the Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests will be employed. Flood loss data for all 51 States which is collected from the Federal Emergency Management Agency (FEMA) is fitted and analyzed. There are samples of 46 observations on the severity of flood loss random variable X from year 1955 to 2000. Years with no flood loss is represented with a small amount, which is 0.1(in millions). In addition we will be interested in testing H_0: Samples come from the postulated probability distribution, as against H_1: Samples do not come from the postulated probability distribution.

6.4 Kolmogorov-Smirnov goodness of fit test

The Kolmogorov-Smirnov (K-S) test statistic is estimated as the largest absolute difference between the cumulative distribution function of the sample data and the cumulative probability distribution function of the proposed probability density function over the range of the random variable:

$$T = \max \left| S_N(x) - F(x) \right|$$

for all x, where $S_N(x)$ is the empirical cumulative distribution function of the sample data and $F(x)$ is the cumulative probability distribution function of the hypothesized probability density function (Lewis 2004). For the K-S test it can be shown that the value of the sample cumulative distribution function is asymptotically normally distributed. This makes the test distribution free which means that the critical values do not depend on the specific probability distribution been tested. For every specified distribution for the null hypothesis, the same set of critical values can be used. The K-S test statistic critical value is approximately $1.224/\sqrt{N}$ or $1.224/\sqrt{46} = 0.1805$ for a 10% level of significance, where N is the total number of observations. For a 5% level of significance it is approximately $1.358/\sqrt{N}$ or 0.2002, and the critical value for a 1% level of significance is approximately $1.628/\sqrt{N}$ or 0.2400.

6.5 Modeling Severity of Flood Losses

Alabama

A distribution is fitted to the flood loss data of the State of Alabama from 1955 to 2000. Table 6.1 shows the statistical characteristics of flood losses (measured in year 2000 dollars) of the State of Alabama. Figure 6.2 illustrates a fitted Weibull distribution against a histogram of the actual data and Table 6.2 presents the K-S goodness-of-fit and the corresponding parameters of each distribution.

Table 6.1 Statistical Characteristics of Flood Losses

Mean	\$68,629,000.00
Median	\$16,305,000.00
Standard deviation	\$163,210,000.00
Skewness	4.6
Kurtosis	23.59

The claim size distributions, especially describing property losses, are usually heavy-tailed. In spite of the fact that one may always work with empirical distribution function derived from a data set of claims, there is always a natural desire to fit a probability distribution with reasonably good mathematical properties to such a data set. In any attempt to do so, one initially performs some exploratory analysis of the data and makes use of descriptive statistics (such as the mean, median, standard deviation, skewness, and kurtosis) and plots. All distributions are fitted using EasyFit 5.4 Professional which undertakes Maximum Likelihood Estimation (MLE). We then try to fit one of the classic parametric distributions using maximum likelihood method to estimate parameters. Various test (for example, the Kolmogorov-Smirnov, Anderson-Darling) may be used to assess the fit of a proposed model.

First, the mean of the sample data is significantly larger than that of the median, which is reflected in a coefficient of skewness equal to 4.63. Second, the losses are very fat tailed, with an excess kurtosis of 23.

Table 6.2 Goodness-of-fit and distribution parameters (Alabama) using K-S statistic.

Distribution	α=0.01	α=0.05	α=0.1	K- S Statistic	Rank	Reject/Accept	Parameters
Standard Beta	0.2400 ***	0.2002	0.1805	0.33426	5	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.10487 $\alpha_{2=}$ 1.5658
Frechet	0.2400	0.2002	0.1805	0.22004	3	Reject H_0 at α=0.05 & 0.1& fail to reject H_0 at α=0.01	α=0.42591 β=2.1108
Gamma	0.2400	0.2002	0.1805	0.25086	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.17681 β=388.15
Gumbel	0.2400 ***	0.2002 **	0.1805 *	0.38212	7	Reject H_0 at α=0.01, 0.05 &0.1	σ=127.26 μ=-4.8256
Lognormal	0.2400	0.2002	0.1805	0.16234	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5692 μ=2.1364
Normal	0.2400 ***	0.2002 **	0.1805 *	0.33729	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=163.21 μ=68.629
Weibull	0.2400	0.2002	0.1805	0.09788	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.46117 β=28.853

***/**/* represent the significance levels at 1%, 5%, and 10%

Table 6.3 Goodness-of-fit and Distribution Parameters (Alabama) using A-D statistic.

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Standard Beta	3.9074	2.5018	1.9286	6.7081	5	Reject H_o α= .01, .05 & .1	$\alpha_{1=}$ 0.10487 $\alpha_{2=}$ 1.5658
	***	**					
Frechet	3.9074	2.5018	1.9286	2.4662	3	Reject Ho at α=0.1	α=0.42591 β=2.1108
	***						
Gamma	3.9074	2.5018	1.9286	3.3092	4	Reject H_o at α =0.1	α= 0.17681 β=388.15
Gumbel	3.9074	2.5018	1.9286	7.2308	6	Reject H_o α= .01, .05 & .1	σ=127.26 μ=-4.8256
	***	**	*				
Lognormal	3.9074	2.5018	1.9286	1.0312	2	Fail to reject H_o at α= .01, .05 & .1	σ=2.5692 μ=2.1364
Normal	3.9074	2.5018	1.9286	8.3944	7	Reject H_0 at α=0.01 0.05 &0.1	σ=163.21 μ=68.629
	***	**	*				
Weibull	3.9074	2.5018	1.9286	0.44237	1	Fail to reject H_0 at α=0.01 0.05 &0.1	α=0.46117 β=28.853

***/**/* represent the significance levels at 1%, 5%, and 10%

Figure 6.1 Weibull probability plot of flood losses for Alabama.

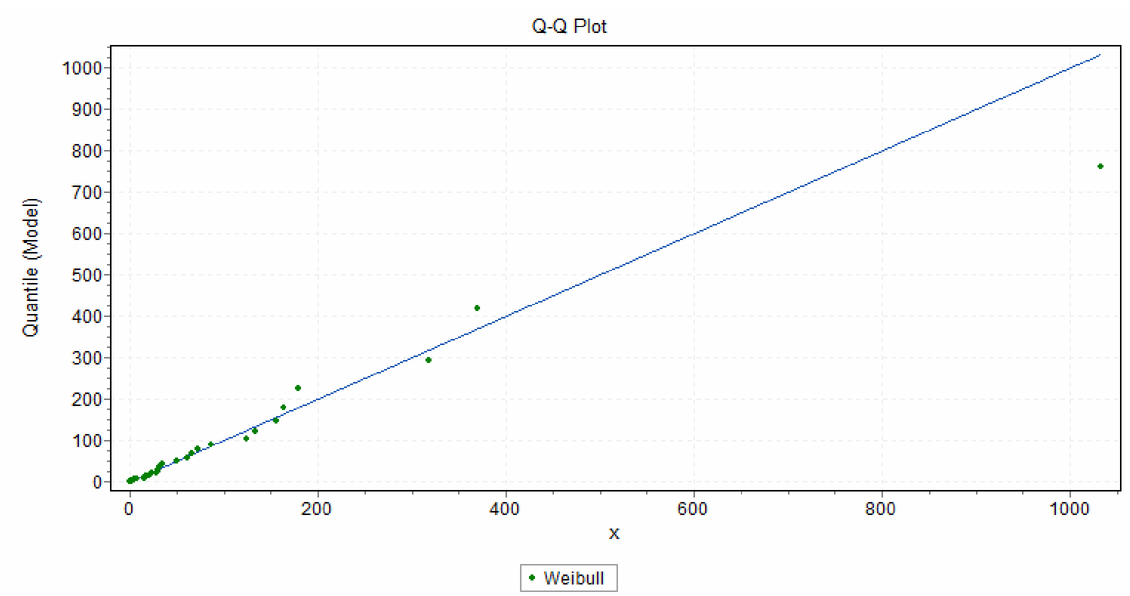
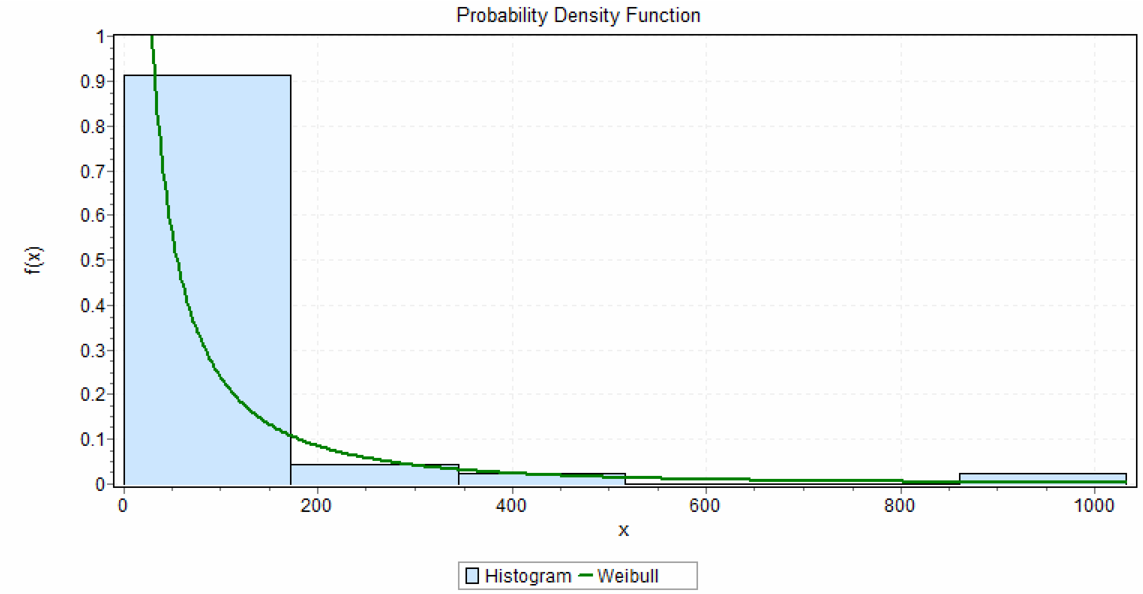


Figure 6.2 Fitted Weibull distribution and histogram for Alabama.



The Weibull distribution was fitted to the flood loss data as shown in Figure 6.2. The standard Beta, Frechet, Gamma, Gumbel, Log-normal, Normal and Weibull were analyzed. This is because in the actuarial literature for describing such claims, continuous distributions are often proposed. The goodness-of-fit was checked with the help of the Kolmogorov-Smirnov and Anderson-Darling test statistic. In the case of the Kolmogorov-Smirnov, the test statistic was compared with the critical value of 0.2400, 0.2002 and 0.1805 with a corresponding 1%, 5% and 10% level of significance. With a 1%, 5% and 10% level of significance the critical value of the Anderson-Darling test statistic was 3.9074, 2.5018 and 1.9286. The Weibull distribution with parameters α=0.46117 and β=28.853 was the best fit for both the Kolmogorov-Smirnov and Anderson-Darling test statistic as shown in Tables 6.2 and 6.3. The log-normal distribution with parameters σ=2.5692 and μ=2.1364 was the next best fit. We can therefore postulate that the data comes from a Weibull distribution and will therefore fail to reject H_0 at $\alpha = 0.01, 0.05$, and 0.1. This is confirmed in the probability plot of Figure 6.1 for which the Kolmogorov-Smirnov test statistic is 0.09788. Table 6.2 appears to indicate that the Weibull distribution fits the data at least as well as the lognormal distribution. The Frechet distribution with parameters α=0.42591 and β=2.1108 fitted well in the case of the Anderson-Darling test statistic with α = 0.01 and 0.05 but for that of the Kolmogorov-Smirnov it fitted with α = 0.01. This indicates that extreme value distributions be considered when fitting flood loss data.

Similar statistical test was conducted for all the remaining 49 States (Alaska – Wyoming) and these can be found at appendix B.

6.6 Applications of the GB2 Distribution in Modeling Insurance Loss Processes

Having talked about the distributions commonly employed in the insurance industry, I will discuss the four parameter generalized beta distribution of the second kind (GB2), which can also be employed to model insurance data.

One of the troubles associated with combined risk theory analysis is the calculation of the cumulative distribution of total annual aggregate losses, *F(x)* for the variable

$$X = \sum_{i=1}^{\hat{N}} \hat{S}_i$$

Where both $\hat{N}$ (number of claims) and $\hat{S}$ (the dollar amount of losses for i^{th} claim) are random variables.

Trying to develop a method in the computation of *F(x)* has been a more complicated approach to the estimation of frequency ($\hat{N}$) and severity ($\hat{S}$) distributions.

The calculation of *F(x),* using the traditional approach was to estimate one or two parameter distribution. Because insurance claims distributions are often heavy-tailed, restricting the set of candidate distributions can also lead to serious underestimation of tail fractals, reinsurance premiums, and other variables (Cummins and Friefelder 1978).

Progress made in modern times has come a long way to open up a much wider range of probability distributions for use in modeling insurance claims processes. Hogg and Klugman (1983) discuss many alternative models for loss distributions as well as related issues of estimation and inference. Cummins *et al* (1978) suggest utilizing the generalized Beta as one of the distributions that is flexible enough to accommodate diverse loss distributions in insurance. Although not implemented in this report, I will discuss this below.

McDonald (1984) considers generalizations of the beta distribution of the first type (Pearson type I) and of the second type (Pearson type VI) which will be denoted by GB1 and GB2, respectively. Venter (1983) introduced the GB2 in the actuarial literature as the transformed beta.

The density function of the generalized beta four parameter distribution (GB2) is given by:

$$\text{GB2(x; a, b, p, q)} = \frac{|a|\, x^{ap-1}}{b^{ap} B(p,q)(1+\{\ \frac{x}{b}\ \}^{a})^{p+q}}, \quad \text{x} > 0$$

where all four parameters *a*, *b*, *p*, *q* are positive. Here *b* is a scale and *a*, *p*, *q* are shape parameters.

The GB2 gives an extremely flexible functional form that can be used to model highly skewed loss distributions especially those seen in non-life insurance. We can make use of the GB2 family of distribution whether the data is untransformed or with natural logs. With data symbolized by extremely heavy trails, the log-GB2 may be better in some instances. The aggregate claims distribution model is based on the moments of a time series of observed total losses. To obtain *F (x)*, a simulation will have to be run from the underlying frequency and best-fitting severity distributions.

The shape and location of the density function is determined by the parameters in these distributions. The parameter b is a scale factor; b is also an upper or lower bound for GB1 variables as the parameter a is positive or negative. GB1 is mentioned in risk theory, but GB2 has no upper limit and therefore it is possible to be used for severity distributions and other applications involving risk theory where the upper tail has no hypothetical boundary. When a is positive, moments of all positive integer orders are defined.

Models provided by GB2 have distributions that portray thick tails. The density and parameters have a complex relationship, but as the value of the parameters a or q become larger, the thinner the tails of the density function. In determining the skewness of the distribution, it is vital to know the relative values of p and q. The GB2 permits positive as well as negative skewness.

The GB2 includes the Log-T (LT), Generalized Gamma (GG), Beta of the 2^{nd} kind (B2), Burr types 3 and 12 (BR3 and BR12), log-Cauchy (LC), Lognormal (LN), Gamma (GA), Weibull, (W), Lomax (L), Fisk, Rayleigh (R), and exponential (EXP) as special or limiting cases. For (x)>0, restriction, a log transformation is helpful, for distributions that are heavily tailed. Insurance data often encounters problems because of its heterogeneity in nature and frequently results in distributions with thick tails. Hogg and Klugman (1983) indicate how mixture distribution provides an approach to modeling unobservable heterogeneity. The GB2 gives an interpretation of the mixture distribution which allows but does not require heterogeneity. The GB2 came as a result of a structural distribution which is $GG(x;a,\theta,p)$ where the scale parameter θ is distributed as a $GG(\theta;-a,b,q)$. Each special case of the GB2 can be interpreted as a mixture. An example is the Log-T which has been shown to be a Lognormal mixed with an inverse Gamma (Cummins and Friefelder 1978, and Hogg and Klugman 1983).

The reason why the GB2 is important in the theory of risk is because it has great flexibility due to the availability of four parameters to model losses.

Hence the GB2 can be justified to have a representation of claims arising from heterogeneous population of exposures. The GB2 distribution can also be obtained from the *F* distribution by making use of the transformation:

$$y = (\frac{p}{q})(\frac{x}{b})^a.$$

If x is GB2, then y will be F with degrees of freedom $d_1 = 2p$ and $d_2 = 2q$.

Chapter 7: Application Scenarios for Single, Mixture, and Kernel Density Distribution

After the application of goodness of fit tests for several distributions to each of the fifty states, there were 4 instances where exactly one distribution fit well (one clear winner), 30 instances where multiple distributions fit adequately (multiple winning competitors), and 16 instances where no parametric distribution fitted adequately (no winner). Mixture distributions are applied when multiple distributions fit adequately, and the non-parametric method of kernel density estimator (KDE) is applied when no distribution fits adequately. Expected values and premium for flood loss claims must be calculated for each of the three scenarios of single, mixture, and kernel density distributions.

Following are examples of the calculations for each scenario.

7.1 Single Distribution (Clear winner)

Texas

Since the Weibull distribution was the clear winner in the state of Texas, we will calculate its expected value followed by the premium of the state. This is given by

$$E(X) = \beta\Gamma(1+\frac{1}{\alpha}) = 314.63\times\Gamma(1+\frac{1}{0.4733})$$

$$= 314.63\times\Gamma(3.1128)$$

$$= 314.63 \times 2.2249$$

$= 700.0203$ (multiply by 1,000,000 to get the mean loss)

$= \$700,020,300$

$$\text{Premium} = \frac{(C+O)(M+1)}{n} = \frac{(\$700,020,300)(1.02)}{674,265}$$

$$\text{Premium} = \$1059.00$$

7.2 Mixture Distributions

Mixture distributions are distributions put together to give the best type of models. A mixture distribution has its shortfalls as Tarpey *et al* (2008) states that a common problem in statistical modeling is to first distinguish between finite mixture distributions and a homogenous non-mixture distribution. This Tarpey *et al* (2008) said was because finite mixture models are widely used in practice and often mixtures of normal densities are indistinguishable from homogenous non-normal densities.

Testing the fit of finite mixture models is a different task, since asymptotic results on the distribution of likelihood ratio statistics do not hold; for this reason, alternative statistics are needed (Revuelta 2008). In insurance the number of claims is often from a Poisson-based discrete distribution whilst individual claim sizes are from a continuous right skewed distribution. The resulting distribution of total claim size is a mixed discrete-continuous model, with positive probability of a zero claim (Heller *et al* 2007).

In modeling the flood loss in United States, classical parametric distribution may not be appropriate, hence the need to fall on a mixture of several different distributions that best explains the data spread. To do this, one may consider mixture modeling which consists of more than one distribution function that explains the data. I will discuss theoretical aspects of mixture distributions below.

If F_1 and F_2 are two distribution functions and p is the weight on distribution 1 and q=1-p on distribution 2, then p: q mixture of F_1 and F_2 has the distribution function F defined by

$$F(x) = pF_1(x) + qF_2(x) \qquad \text{(Boland 2007)}$$

where X, X_1, and X_2 are random variables with respective distributions F, F_1, and F_2, then we say that X is a p: q mixture of the random variables X_1 and X_2. In theory one can form mixtures of many types of random variables which lead to very complicated distributions. Consider the amount of flood loss from different States in the U.S where the random variable X represents the number of annual flood loss claims arising as a result of the flood loss from a randomly selected policyholder. Since distribution expresses the probability of a number of events (in this case floods) occurring in a fixed period of time with a known average rate and independently of the time from the last event, then our random variable X is often modeled as a Poisson distribution with parameter λ, where λ is the flood loss claim rate.

Conditional on knowing the flood loss claim rate λ, which is not constant among policyholders, one might assume that the possibilities for λ vary over $(0,\infty)$ according to some probability distribution. For λ, the most attractive distribution is Gamma distribution. Gamma distribution is frequently used as a probability model for waiting times. In this case, the waiting time is the time between average claims filed by two policy holders. This waiting time is a random variable that follows a gamma distribution.

If $X \sim P(\lambda)$ and $\lambda \sim \Gamma(\alpha,\beta)$, then the mixture distribution is given by

$$P(X = x) = \int_0^\infty P(X = x \mid \lambda) dG_\wedge(\lambda)$$

$$= \int_0^\infty \frac{\lambda^x e^{-\lambda}}{x!} \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\lambda\beta}}{\Gamma(\alpha)} d\lambda \qquad \text{for } \lambda \succ 0$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha) x!} \int_0^\infty \lambda^{(x+\alpha)-1} e^{-(1+\beta)\lambda} d\lambda \qquad (1)$$

Now consider

$$I=\int_0^\infty \lambda^{(x+\alpha)-1}e^{-(1+\beta)\lambda}d\lambda$$

$$I=\lim_{a\to\infty}\int_0^\infty \lambda^{(x+\alpha-1)}e^{-(\beta+1)\lambda}d\lambda$$

Using integration by parts we have

$$I=\lim_{a\to\infty}-\lambda^{(x+\alpha-1)}\frac{e^{-(\beta+1)\lambda}}{\beta+1}\Big]_0^a+(\frac{x+\alpha-1}{\beta+1})\int_0^\infty \lambda^{x+\alpha-1}e^{-(\beta+1)\lambda}d\lambda$$

$$I=-\lim_{a\to\infty}a^{(x+\alpha-1)}\frac{e^{-(\beta+1)a}}{\beta+1}+(\frac{x+\alpha-1}{\beta+1})\int_0^\infty \lambda^{x+\alpha-2}e^{-(\beta+1)\lambda}d\lambda$$

$$I=(\frac{x+\alpha-1}{\beta+1})\int_0^\infty \lambda^{x+\alpha-2}e^{-(\beta+1)\lambda}d\lambda$$

Now we know that

$$I=\int_0^\infty \lambda^{(x+\alpha)-1}e^{-(1+\beta)\lambda}d\lambda$$

gave us

$$I=(\frac{x+\alpha-1}{\beta+1})\int_0^\infty \lambda^{x+\alpha-2}e^{-(\beta+1)\lambda}d\lambda$$

Using the same approach of integration by parts for the above, we have

$$I=(\frac{x+\alpha-1}{\beta+1})(\frac{x+\alpha-2}{\beta+1})\int_0^\infty \lambda^{x+\alpha-3}e^{-(\beta+1)\lambda}d\lambda$$

Continuing the process to $(x+\alpha-1)^{th}$ integral, we have

$$I=\frac{x+\alpha-1}{\beta+1})(\frac{x+\alpha-2}{\beta+1})(\frac{x+\alpha-3}{\beta+1})\ldots\int_0^\infty e^{-(\beta+1)\lambda}d\lambda$$

Finally for $(x+\alpha)^{th}$ integral, this gives us

$$I = (\frac{x+\alpha-1}{\beta+1})(\frac{x+\alpha-2}{\beta+1})(\frac{x+\alpha-3}{\beta+1})\ldots(\frac{1}{\beta+1})$$

Therefore

$$I = \frac{(x+\alpha-1)!}{(\beta+1)^{x+\alpha}}$$

Rewriting the numerator using the gamma function, we have

$$I = \frac{\Gamma(x+\alpha)}{(\beta+1)^{x+\alpha}} \qquad (2)$$

Putting equation (2) into equation (1), we have

$$P(X=x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)x!}\frac{\Gamma(\alpha+x)}{(\beta+1)^{\alpha+x}}$$

$$= \frac{\Gamma(x+\alpha)}{\Gamma(\alpha)x!}\frac{\beta^{\alpha}}{(\beta+1)^{\alpha}}.\frac{1}{(\beta+1)^{x}}$$

$$= \frac{\Gamma(\alpha+x)}{\Gamma(\alpha)x!}(\frac{\beta}{\beta+1})^{\alpha}(\frac{1}{\beta+1})^{x}$$

$$= \frac{\Gamma(x+\alpha)}{\Gamma(\alpha)x!}p^{\alpha}q^{x} \qquad (3)$$

This follows a negative binomial distribution where $p = \frac{\beta}{\beta+1}$ and $q = \frac{1}{\beta+1}$.

In summary if $X \sim P(\lambda)$ and $\lambda \sim \Gamma(\alpha,\beta)$, then $X \sim NB(\alpha, p)$.The Negative Binomial distribution then best fits the arrival of flood loss data for which the claim rate need not to be constant or homogenous, properties that overshadow Poisson distribution. The Negative Binomial distribution is a two parameter distribution.

I will now calculate the mean and premium for the state of Alabama where the Lognormal and Weibull distributions failed to reject.

In general

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

For mixture distribution of $f_a(x)$ and, $f_b(x)$ f_a is weighted by a factor of w_a, and f_b is weighted by a factor of $w_b = 1 - w_a$.

$$E(X\ ) = \int_{-\infty}^{\infty} w_a x f_a(x)dx + \int_{-\infty}^{\infty} w_b x f_b(x)dx$$

$$= w_a \int_{-\infty}^{\infty} x f_a(x)dx + w_b \int_{-\infty}^{\infty} x f_b(x)dx$$

$$= w_a E(X_a) + w_b E(X_b) \qquad (***)$$

Alabama

Suppose a data set consists of observations $x_1,\ldots, x_n$ from a probability distribution $f(x,\theta_1,\theta_2)$ depending upon two unknown parameters, then the maximum likelihood estimates $\theta_1$and θ_2 are the values of the parameters that jointly maximize the likelihood function

$$L(x_1,\ldots,x_2,\theta_1,\theta_2) = f(x_1,\theta_1,\theta_2)\times\ldots\times f(x_n,\theta_1,\theta_2) \qquad (1)$$

which can be thought of as the "likelihood" of observing the data values $x_1,\ldots,x_n$ for a given value of $\theta_1$and θ_2. To maximize the joint density functions, one will have to take the derivatives of the log-likelihood with respect to $\theta_1$and θ_2 and setting the two resulting expressions equal to zero.

When two distributions *A* and *B* fit $(x_1,\ldots,x_n)$, then the weight on distribution *A* is calculated as

$$w_A = \frac{L_A}{L_A + L_B}$$

where L_A is the value of the maximum for the likelihood function in eqn. (1) for parameter estimates θ_1, and θ_2. Similarly the weight on distribution B will be calculated as

$$w_B = \frac{L_B}{L_B + L_A}$$

where L_B is the value of the maximum for the likelihood function in eqn. (1) for parameter estimates θ_1, and θ_2.

For Alabama two distributions, Weibull and Lognormal fit using the Kolmogorov-Smirnov (K-S) test statistic. JUMP software provides

$$-2\log(L_{Lognormal}) = 414.5060$$

and

$$-2\log(L_{Weibull}) = 412.3174$$

Now calculating the likelihood for the Lognormal and Weibull distributions we have

$$-2\log(L_{Lognormal}) = 414.5060$$

$$\log(L_{Lognormal}) = -207.2530$$

$$L_{Lognormal} = e^{-207.2530} = 1.00114\times 10^{-90}$$

and

$$-2\log(L_{Weibull}) = 412.3174$$

$$\log(L_{Weibull}) = -206.1587$$

$$L_{Weibull} = e^{-206.1587} = 2.99018\times 10^{-90}$$

This gives me the weights

$$w_{Lognormal} = \frac{L_{Lognormal}}{L_{Lognormal} + L_{Weibull}} = \frac{(1.00114\times 10^{-90})}{(1.00114\times 10^{-90}) + (2.99018\times 10^{-90})} = \frac{(1.00114\times 10^{-90})}{(3.99132\times 10^{-90})} = 0.2508$$

and $w_{Weibull} = \dfrac{w_{Weibull}}{w_{Weibull} + w_{Lognormal}} = \dfrac{(2.99018 \times 10^{-90})}{(2.99018 \times 10^{-90}) + (1.00114 \times 10^{-90})} = \dfrac{(2.99018 \times 10^{-90})}{(3.99132 \times 10^{-90})} = 0.7492$

Now, the mean (Loss) from the Lognormal distribution is given by

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} = e^{2.1364 + \frac{(2.5692)^2}{2}} = e^{5.4368} = 229.5765$$

and that of the Weibull distribution is also given by

$$E(X) = \beta\Gamma(1 + \frac{1}{\alpha}) = 28.853 \times \Gamma(1 + \frac{1}{0.4612})$$

$$= 28.853 \times \Gamma(3.1683)$$

$$= 28.853 \times 2.3489$$

$$= 67.7728$$

Therefore mixing the two distributions in this proportion using (***), I can get the expected loss which is given by

$$E\left[X\right] = (0.2508) \times (229.5765) + (0.7492) \times (67.7728)$$

$$= 57.5778 + 50.7754$$

$$= 108.3532$$

= $108,353,200 (multiply by 1,000,000 to get the mean loss)

Assuming that there is no operational cost, O, the premium will be calculated as follows:

$$= \frac{(C + O)(M + 1)}{n}$$

From section 1.2 on page 3 we defined that C is the total expected loss amount, M is the target profit margin, and n is the number of insured entities in the state. Therefore the premium for the state of Alabama is calculated as:

Premium ≈ (108,353,200) × (1.02) / 56908

Premium ≈ \$1,942

7.3 Kernel Density Estimation

Kernel density estimation is a non-parametric approach of estimating the probability density function (*pdf*) of a random variable. In recent times density estimation has been used in many fields, including archaeology (e.g., Baxter, Beardah, and Westwood, 2000), banking (e.g., Tortosa-Ausina, 2002), climatology (e.g., DiNardo, Fortin, and Lemieux, 1996), genetics (e.g., Segal, and Wiemels, 2002), hydrology (e.g., Kim, and Heo, 2002) and physiology (e.g., Paulsen, and Heggelund, 1996). Sheather (2004) has also used this approach to estimating geographic customer densities. In this discussion I intend to use kernel density estimate to illustrate how sample data from the flood loss of New Jersey can be estimated into a continuous probability density function. I will consider a nonparametric approach where less rigid assumptions will be made about the distribution of the observed data. Although it will be assumed that the distribution has a probability density f, the data are allowed to speak for themselves in determining the estimate of f more than would be in the case if f were to fall in a given parametric family.

In estimating a density using the kernel density estimation requires a *kernel function* K and a smoothing parameter h which is also called the bandwidth. The density which is generally estimated will be sensitive to the choice of the kernel function, but may be strongly affected by the value of the bandwidth. A large bandwidth results in a smooth- looking surface, while a small h results in a surface which is bumpier.

If we denote $X_1, X_2, ..., X_n$ to be a sample of size *n* from a random variable with density *f*, then the kernel density estimate of *f* at point *x* is given by (See Sheather 2004)

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x - X_i}{h}) \quad ,$$

where the kernel *K* satisfies $\int K(x)dx = 1$, *n* is the number of data points and *h* is the bandwidth or window width. In this case, *K* satisfies the conditions

$$\int K(y)dy = 1,$$

$$\int yK(y)dy = 1,$$

The Gaussian kernel which is a popular choice for *K* is given by

$$K(y)\frac{1}{\sqrt{2\Pi\sigma}}\exp(\frac{-y^2}{2\sigma^2})$$

A commonly used choice of an overall measure of the inconsistency between the kernel density estimate, $\hat{f}_h$ and density, *f* is the mean integrated square error (MISE), which is given by

$$MISE(\hat{f}_h) = E\left\{\int (\hat{f}_h(y) - f(y))^2 dy\right\}$$

This may be computed for a range of bandwidth and the optimum choice is the width that minimizes this error. Sprent *et al* (2007) explains that there are also some rule of thumb guides for choosing the bandwidth. One of the often used rule of thumbs for bandwidth *h* which minimizes the mean integrated square error is given by

$$h = 1.06s/n^{1/5},$$

where *s* is the sample standard deviation and *n* is the number of data points. This bandwidth selection is based on Gaussian kernel. There are different softwares for calculating the kernel density estimates. I used the R software which produces kernel density estimates with a default kernel the Gaussian density with mean 0 and standard deviation 1.

Figure 7.1. Kernel density estimates based on the bandwidth that minimizes the mean integrated square error.

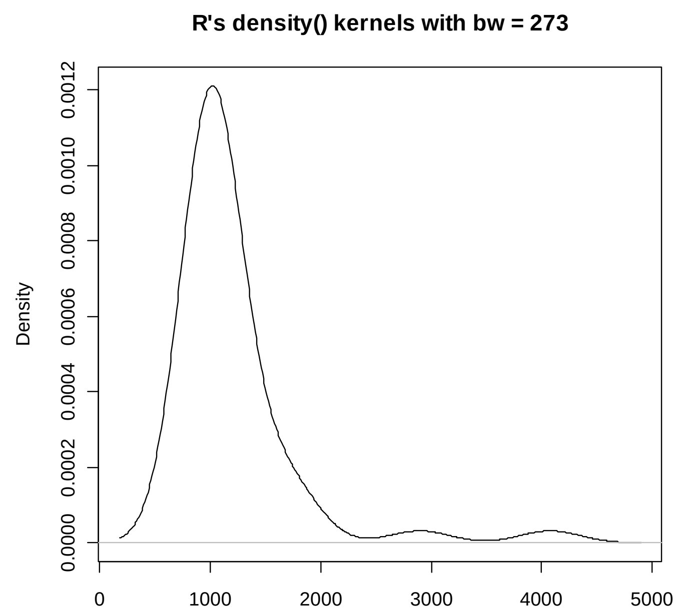
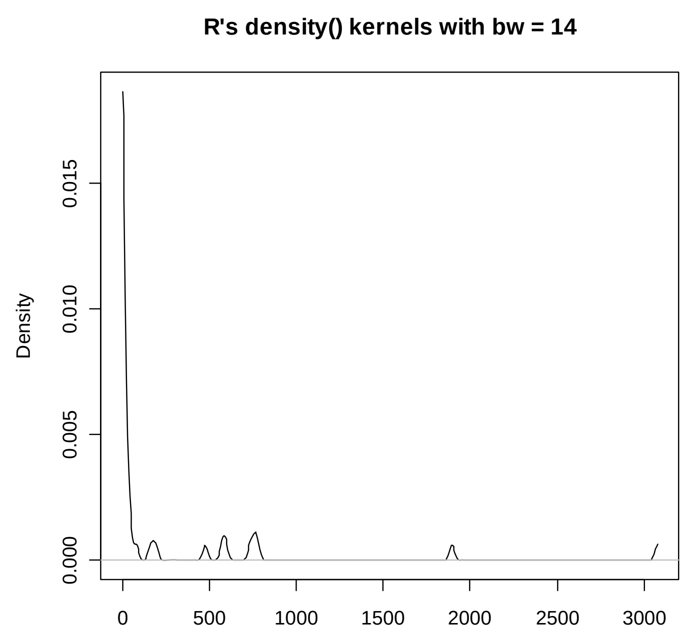


Figure 7.2. Kernel density estimates with under smooth graph.



I will use the flood loss of New Jersey to demonstrate the kernel density estimation choosing an appropriate bandwidth, h. Figure 7.1 shows Gaussian kernel density estimates based on the bandwidth which minimizes the mean integrated square error (MISE). When this method is implemented, R gives the graph as shown above. We will briefly review different methods for choosing a value of the bandwidth, h. The bandwidth that generates a good consistent estimate of the Gaussian kernel is given by

$$h_{mise} = 1.06\sigma n^{-1/5} \qquad \text{(Silverman 1985)}$$

where σ denotes the standard deviation and n is the sample size. From the flood loss of New Jersey, $\sigma = 533.18$, $n = 46$ and so

$$h_{MISE} = 1.06(533.18)(46)^{-1/5} = 273$$

Sometimes Silverman rule of thumb (SROT) bandwidth is applied. It is given by

$$h_{SROT} = 0.9An^{-1/5} \qquad \text{(Silverman 1985)}$$

where A is the sample interquartile range/1.34 and n is the number of data points. Application of this rule yields a bandwidth of $h = 14$ which does not produce a smooth curve as can be seen in Figure 7.2.

I will now calculate the mean and premium for the state of Hawaii and use the Riemann sum approach to show that the area under the curve is one which is a condition of a kernel density.

Riemann Sum Application:

The Riemann sum calculates the sum of the area of rectangles whose weight is defined by the value of a given function at the boundary point of each sub interval. Let f be defined on a closed interval $[a, b]$, and let P be a partition of $[a, b]$. A Riemann sum of $f(x)$ for P is any expression R_p of the form

$$R_p = \sum_{k=1}^{n} f(w_k)\Delta x_k, \qquad \text{(Swokowski } et\ al \text{ 1994)}$$

where w_k is in $[x_{k-1}, x_k]$ and $k = [1, 2,\ldots, n]$. Let Δx_k be defined as

$$\Delta x_k = x_k - x_{k-1}$$

We have observed that the density function in R produces uniform values for Δx_k, so

$$x_k - x_{k-1}, (\Delta x),$$
$$x_k - x_{k-1} = \frac{x_n - x_0}{n}$$

Then

$$R_p = \Delta x_k \sum_{k=1}^{n} f(w_k)$$

Now, since

As $n \rightarrow \infty$, $x_k \rightarrow 0$, and R_p becomes an approximation of the integral

$$\int_a^b f(x)dx$$

where $a = x_0$ and $b = x_n$ (the end-points of the interval $[a,b]$).

The integral over the range of a true probability density function (*pdf*) should be 1. Thus, a Riemann sum of a Kernel Density Estimator (KDE) function should sum to 1. The KDE generated by the R software was checked and validated that this was true. The default output given by R estimates the density (y) at each of 512 equally spaced points (x), based on a range of the input data set. In this case with the R software, n was 512, and

$$\Delta x_k = \frac{\max(x) - \min(x)}{512}$$

Hawaii

Hawaii is an example where none of the tested parametric distributions fit the data adequately. Hence, a KDE function was used as the non-parametric approximation of this distribution. The Riemann sum was used to validate that the KDE curve generated by the software integrates to1. Let $F(w_k) = F(x_k) = y_k$, the probability that the flood loss amount will be between x_{k-1} and x_k. This gives

$$R_{Hawaii} = \sum_{k=1}^{512} \Delta x_k y_k = 1.000207$$

R was used to compute the kernel density for the state of Hawaii. If the kernel density estimation is done on the raw flood loss data, the KDE has positive density for negative flood loss values. But, in reality, there are never negative flood losses. To solve for this difficulty, a log transformation was utilized. Let v be the raw flood loss data.

$$z = \ln(v)$$

The KDE was generated on z in the R software. The log transformation was then reversed using

$$x = e^z$$

This, of course, means x_i is never less than zero, as desired.

Now we can calculate the expected value as a probability mass function by multiplying each observation by y, and summing these products to get an estimate of the mean. It is given by the formula

$$E[X] = \sum_{x:p(x)>0} xp(x) \qquad \text{(Ross 2007)}$$

For purposes of using the numerical approximation produced by the software, treat the approximation as a probability mass function, where the probability is uniform for each interval $x_{i-1}<x<x_i$. Then the expected flood loss amount

$$C{+}O = \sum_{i=1}^{n} x_i\, p(x_{i-1} < x < x_i) \ ,$$

where $p(x_{i\text{-}1}<x<x_i) = p(z_{i\text{-}1}<z<z_i)$.

Thus, expected flood loss amount, using a log transformation and kernel density method, is $37,584,220.

For Hawaii the number of policy holders, n, is 59336, and the margin M is assumed to be 0.02.

The premium per policy per year is therefore calculated as

$$Premium = \frac{(C+O)(M+1)}{n} = \frac{(\$37,584,220)(1.02)}{59336} = \$646.00$$

Chapter 8: Conclusions and Future Study

This project has investigated what distribution(s) fit the flood loss data covering all the 50 States of the United States of America from 1955 to 2000, using the Kolmogorov-Smirnov test statistic and Anderson-Darling test statistic. Based on the analysis performed, the following conclusions can be made.

- Considering the distributions using the Kolmogorov-Smirnov statistic, the following were the statistics. The Weibull distribution was ranked first 25 times, followed by the Frechet distribution. The standard Beta and Lognormal distributions which were jointly ranked third appeared 5 times each. This was followed by the gamma distribution with fitted losses from 4 States.
- On the other hand the Anderson-Darling statistic had the following ranking for the distributions. The Weibull distribution came first appearing 25 times followed by the Frechet distribution appearing 18 times. The lognormal distribution was ranked third fitting well to eight States with Gamma and Beta distributions coming forth and fifth with 3 and 1 respectively.
- The Weibull and Frechet distributions had a good fit compared to the Gumbel, Normal, Lognormal, Gamma, and standard Beta distributions making extreme value distributions suitable for fitting flood losses, even though it did not fit well for some States.

- There are noticeable correlations among some adjacent states. This means that flooding in one State can result in flooding in another neighboring State.

Appendix A

Severity Distributions:

Beta (α, β)

pdf $$f(x/\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1, \quad \alpha > 0, \quad \beta > 0$$

mean and variance $$EX = \frac{\alpha}{\alpha+\beta}, \quad VarX = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

skewness $$\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$$

Gamma (α, β)

pdf $$f(x/\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha, \beta > 0$$

mean and variance $$EX = \alpha\beta, \quad VarX = \alpha\beta^2$$

skewness $$\frac{2}{\sqrt{\alpha}}$$

Lognormal (μ, σ^2)

pdf $$f(x/\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

mean and variance $$EX = e^{\mu + (\sigma^2/2)}, \quad VarX = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$$

skewness $$(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$$

Normal (μ, σ^2)

pdf $$f(x/\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

mean and variance $$EX = \mu, \quad VarX = \sigma^2$$

skewness $$0$$

Pareto (α, β)

pdf $$f(x/\alpha,\beta)=\frac{\beta\alpha^{\beta}}{x^{\beta+1}}, \quad a<x<\infty,\ \alpha>0,\quad \beta>0$$

mean and variance $$EX=\frac{\beta\alpha}{\beta-1},\quad \beta>1, \qquad VarX=\frac{\beta\alpha^{2}}{(\beta-1)^{2}(\beta-2)},\quad \beta>2$$

skewness $$\frac{2(1+\alpha)}{(\alpha-3)}\sqrt{\frac{\alpha-2}{\alpha}},\quad \alpha>3$$

Weibull (γ, β)

pdf $$f(x/\gamma,\beta)=\frac{\gamma}{\beta}x^{\gamma-1}e^{-x^{\gamma}/\beta},\quad 0\le x<\infty,\quad \gamma>0,\quad \beta>0$$

mean and variance $$EX=\beta^{1/\gamma}\Gamma(1+\frac{1}{\gamma}),\quad VarX=\beta^{2/\gamma}\left[\Gamma(1+\frac{2}{\gamma})-\Gamma^{2}(1+\frac{1}{\gamma})\right]$$

Frequency Distributions:

Binomial (n, P)

pmf $$P(X=x/n,p)=\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x};\quad x=0,1,2,\ldots,n;\quad 0\le p\le 1$$

mean and variance $$EX=np,\quad VarX=np(1-p)$$

skewness $$\frac{1-2p}{\sqrt{np(1-p)}}$$

Negative binomial (r, p)

pmf $$P(X=x/r,p)=\frac{(r+x-1)!}{x!(r-1)!}p^{r}(1-p)^{x};\quad x=0,1,\ldots;\quad 0\le p\le 1$$

mean and variance $$EX=\frac{r(1-p)}{p},\quad VarX=\frac{r(1-p)}{p^{2}}$$

skewness $$\frac{2-p}{\sqrt{r(1-p)}}$$

Poisson (λ)

pmf	$P(X = x / \lambda) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}; \quad x = 0,1,...; \quad 0 \le \lambda < \infty$
mean and variance	$EX = \lambda, \quad VarX = \lambda$
skewness	$q^{-1/2}$

Appendix B

Table B.1 Goodness-of-fit and Distribution Parameters (Alaska)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.28149	1	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1}=$ 0.00312 $\alpha_{2}=$ 0.12486
Frechet	0.2400	0.2002	0.1805	0.35423	3	Reject H_0 at α=0.01,0.05 & 0.1	α=0.49141 β=0.10785
Gamma	0.2400	0.2002	0.1805	0.77371	7	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.02789 β=1720.6
Gumbel	0.2400	0.2002	0.1805	0.49894	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=224.05 μ=-81.334
Lognormal	0.2400	0.2002	0.1805	0.40305	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.3958 μ=0.92416
Normal	0.2400	0.2002	0.1805	0.47757	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=287.35 μ=47.99
Weibull	0.2400	0.2002	0.1805	0.34688	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.31815 β=1.543

Figure B.1 Fitted beta distribution and histogram for Alaska

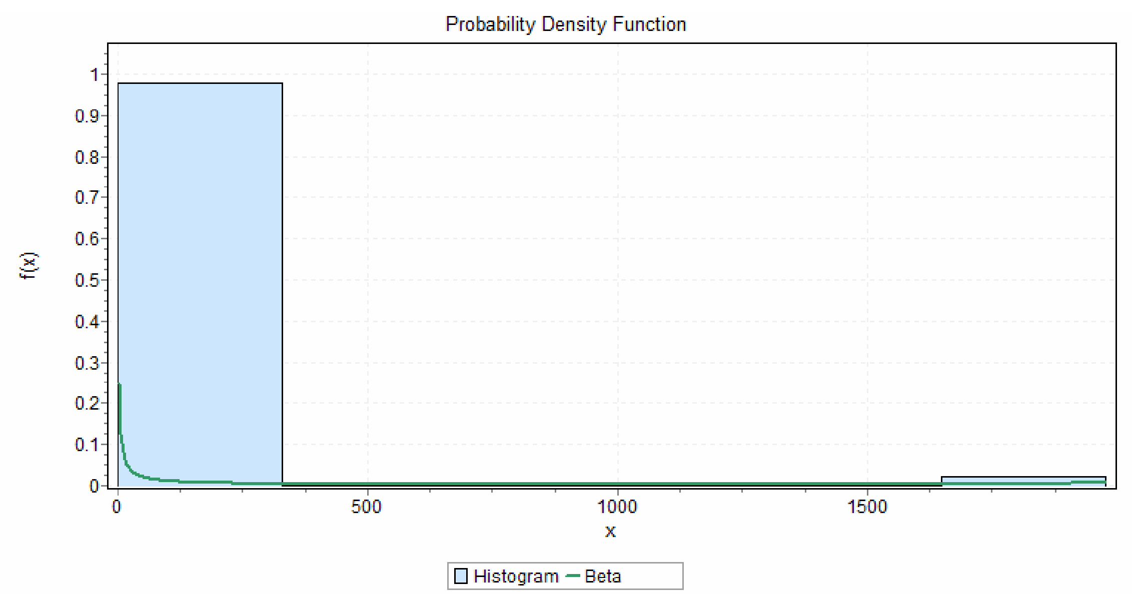


Table B.2 Goodness-of-fit and distribution Parameters (Arizona)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.33509	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.10565 $\alpha_{2=}$ 0.85215
	***	**	*				
Frechet	0.2400	0.2002	0.1805	0.1786	3	Fail to reject H_0 at α=0.01,0.05 & 0.1	α=0.36528 β=0.59276
	***	8**	*				
Gamma	0.2400	0.2002	0.1805	0.17579	2	Fail to reject H_0 at α=0.01,0.05 & 0.1	α= 0.20053 β=329.58
Gumbel	0.2400	0.2002	0.1805	0.36898	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=115.07 μ=-0.33213
	***	**					
Lognormal	0.2400	0.2002	0.1805	0.19552	4	Reject H_0 at α=0.1	σ=3.0796 μ=1.0949
Normal	0.2400	0.2002	0.1805	0.32827	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=147.59 μ=66.09
	***	**	*				
Weibull	0.2400	0.2002	0.1805	0.17377	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.3514 β=14.039

Figure B.2 Fitted Weibull distribution and histogram for Arizona

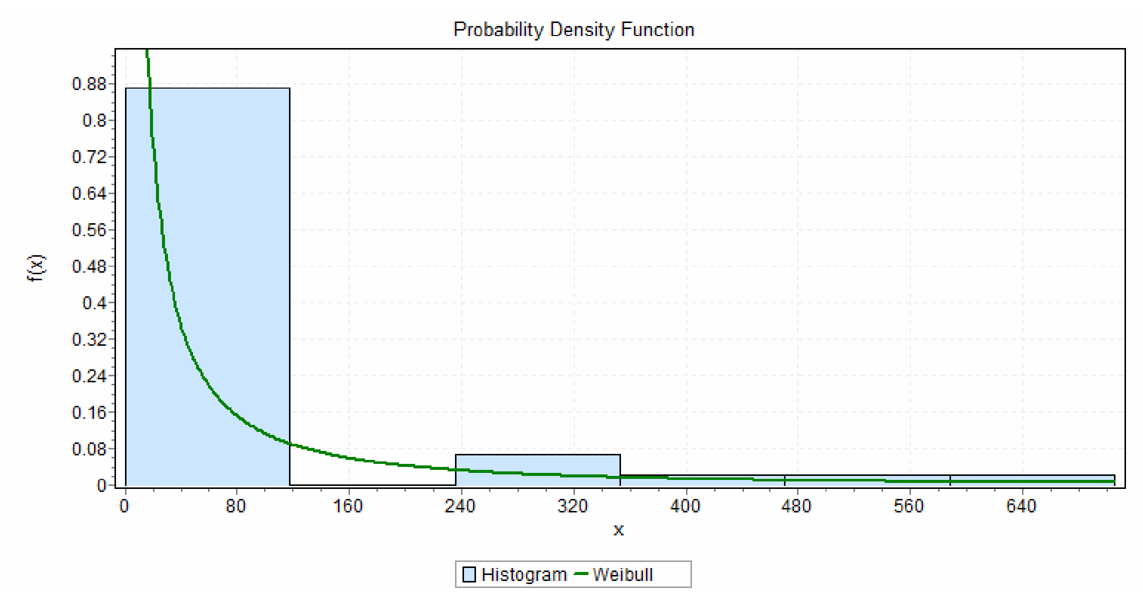


Table B.3 Goodness-of-fit and Distribution Parameters (Arkansas)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.40666	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.07626 $\alpha_{2=}$ 0.74122
Frechet	*** 0.2400	** 0.2002	* 0.1805	0.14521	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.44867 β=2.9167
Gamma	0.2400	0.2002	0.1805	0.26938	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.15501 β=748.1
Gumbel	0.2400	0.2002	0.1805	0.3946	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=229.65 μ=-16.595
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.07713	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5344 μ=2.3967
Normal	0.2400	0.2002	0.1805	0.34702	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=294.53 μ=115.96
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.07339	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.43347 β=37.938

Figure B.3 Fitted Weibull distribution and histogram for Arkansas

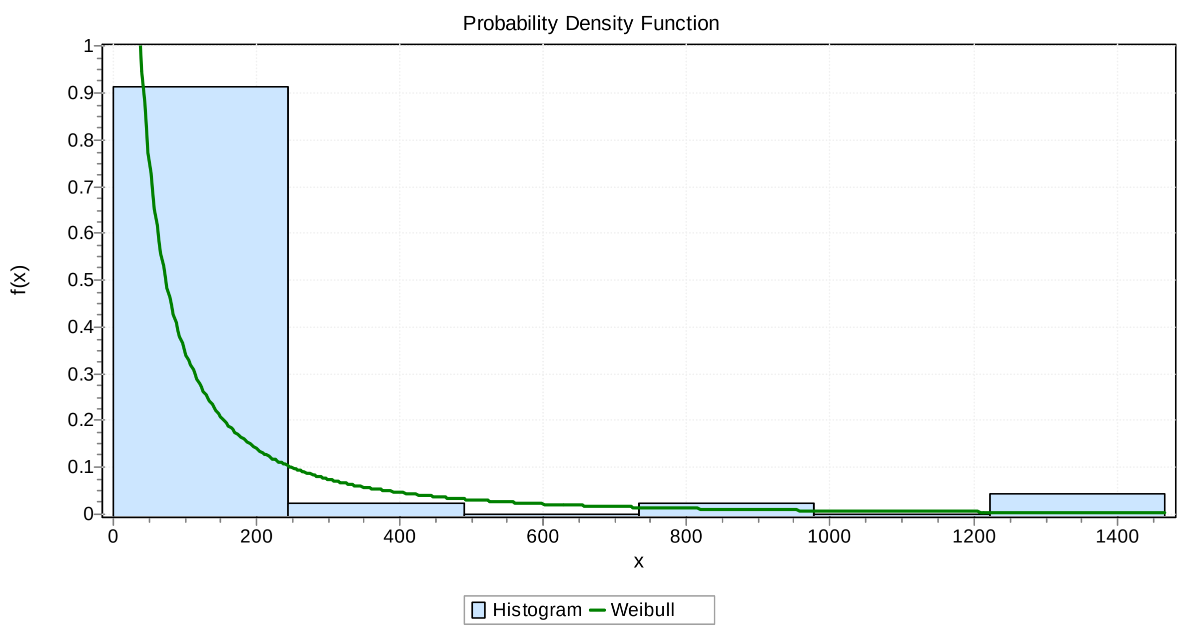


Table B.4 Goodness-of-fit and Distribution Parameters (California)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	** 0.2002	0.1805	0.1919	4	Fail to Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.12378 $\alpha_{2=}$ 1.2044
Frechet	*** 0.2400	0.2002	0.1805	0.22339	5	Reject H_0 at α=0.01,0.05	α=0.30267 β=6.7467
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.09601	1	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.26245 β=2954.6
Gumbel	0.2400	0.2002	0.1805	0.33858	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=1180.2 μ=94.21
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.1669	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.494 μ=3.8289
Normal	0.2400	0.2002	0.1805	0.30425	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=1513.6 μ=775.42
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.09647	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36534 β=245.13

Figure B.4 Fitted Weibull distribution and distribution for California

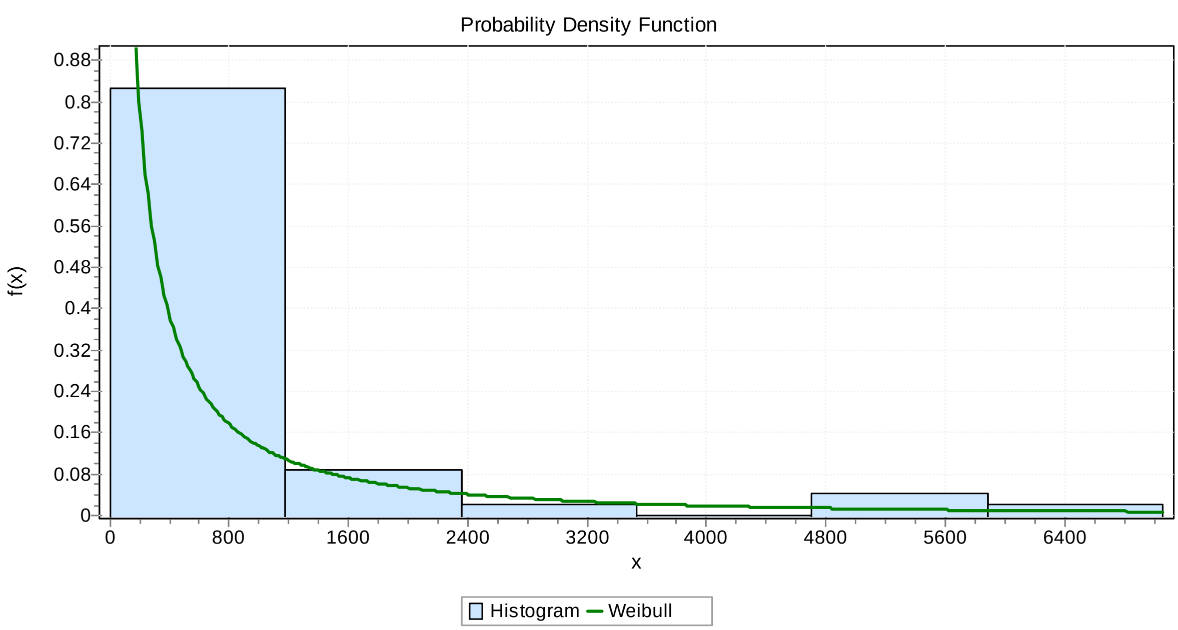


Table B.5 Goodness-of-fit and Distribution Parameters (Colorado)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.80303	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.10487 $\alpha_{2=}$ 1.5658
Frechet	*** 0.2400	** 0.2002	* 0.1805	0.17958	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36624 β=0.36258
Gamma	0.2400	0.2002	0.1805	0.60995	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.17681 β=388.15
Gumbel	0.2400	0.2002	0.1805	0.48051	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1163.1 μ=-361.4
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.17053	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5692 μ=2.1364
Normal	0.2400	0.2002	0.1805	0.4177	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=163.21 μ=68.629
Weibull	*** 0.2400	0.2002	0.1805	0.21514	3	Fail to Reject H_0 at α=0.01 & reject H_0 at α=0.05 & 0.1	α=0.46117 β=28.853

Figure B.5 Fitted lognormal distribution and histogram for Colorado

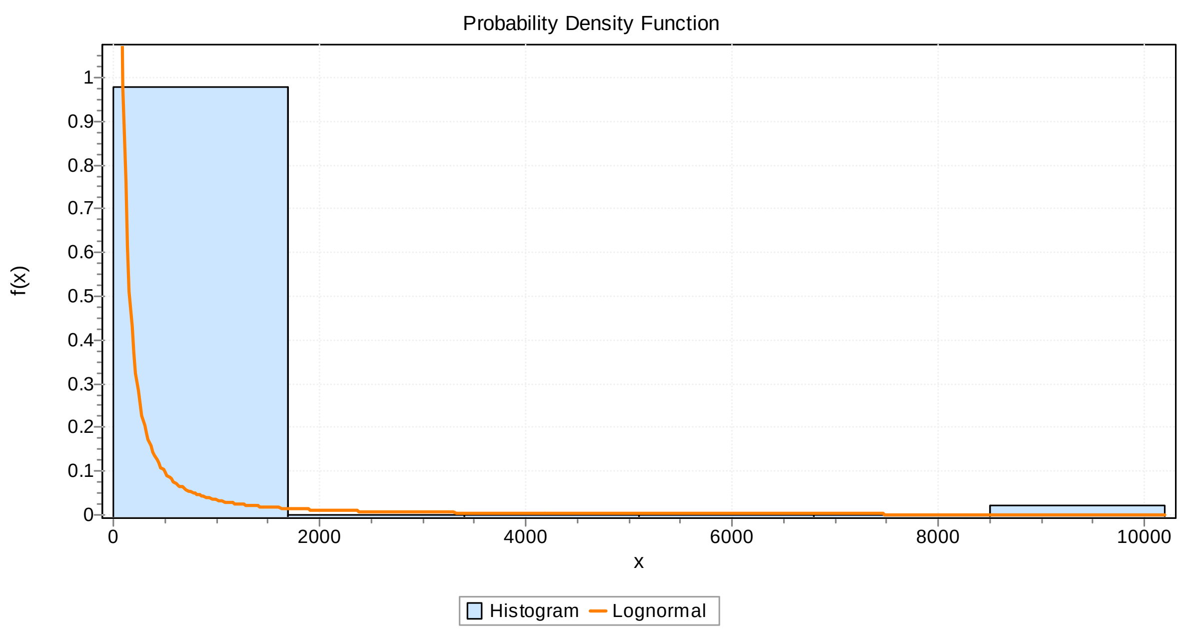


Table B.6 Goodness-of-fit and Distribution Parameters (Connecticut)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.93198	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.0014 $\alpha_{2=}$ 0.05033
Frechet	0.2400	0.2002	0.1805	0.30263	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.42308 β=0.12988
Gamma	0.2400	0.2002	0.1805	0.72961	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.0243 β=11237.0
Gumbel	0.2400	0.2002	0.1805	0.50373	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1365.7 μ=-515.28
Lognormal	0.2400	0.2002	0.1805	0.35831	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.9226 μ=0.52569
Normal	0.2400	0.2002	0.1805	0.4961	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=1751.6 μ=273.05
Weibull	0.2400	0.2002	0.1805	0.30371	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.27401 β=2.9717

Figure B.6 Fitted Weibull distribution and histogram for Connecticut

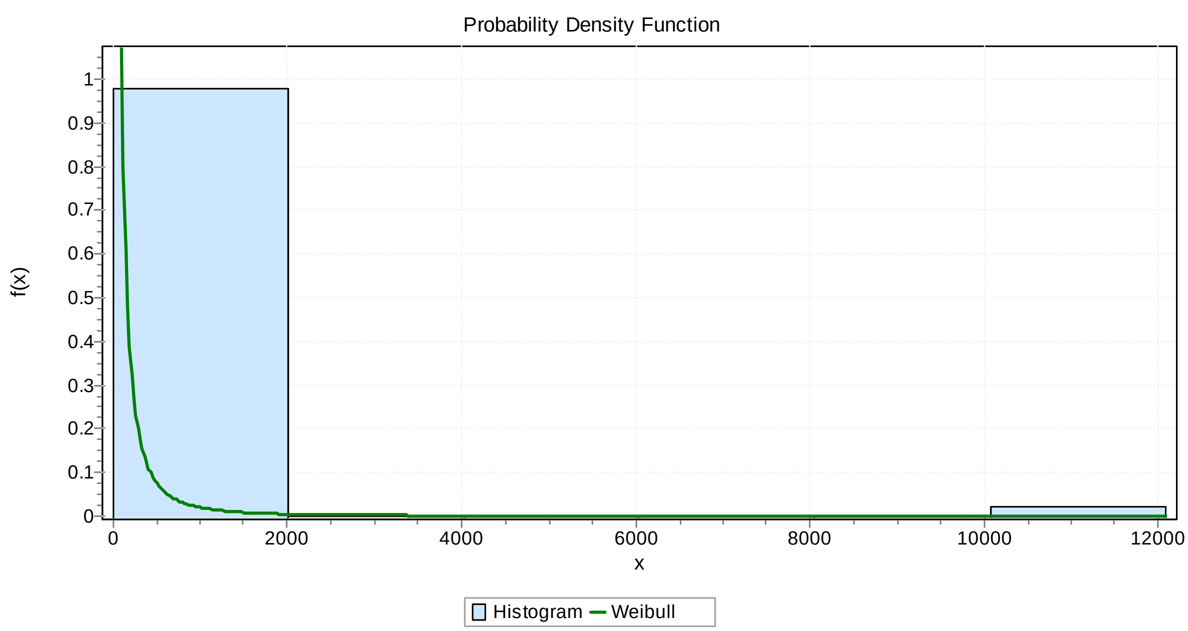


Table B.7 Goodness-of-fit and Distribution Parameters (Delaware)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.76594	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.0373$\alpha_{2=}$ 0.69685
Frechet	0.2400	0.2002	0.1805	0.44003	2	Reject H_0 at α=0.01,0.05 & 0.1	α=1.0171 β=0.07304
Gamma	0.2400	0.2002	0.1805	0.69142	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08564 β=5.967
Gumbel	0.2400	0.2002	0.1805	0.44861	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.3615 μ=-0.27486
Lognormal	0.2400	0.2002	0.1805	0.46513	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.0792 μ=-1.9796
Normal	0.2400	0.2002	0.1805	0.4609	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.7462 μ=0.51104
Weibull	0.2400	0.2002	0.1805	0.42787	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.54953 β=0.21479

Figure B.7 Fitted Weibull distribution and histogram for Delaware

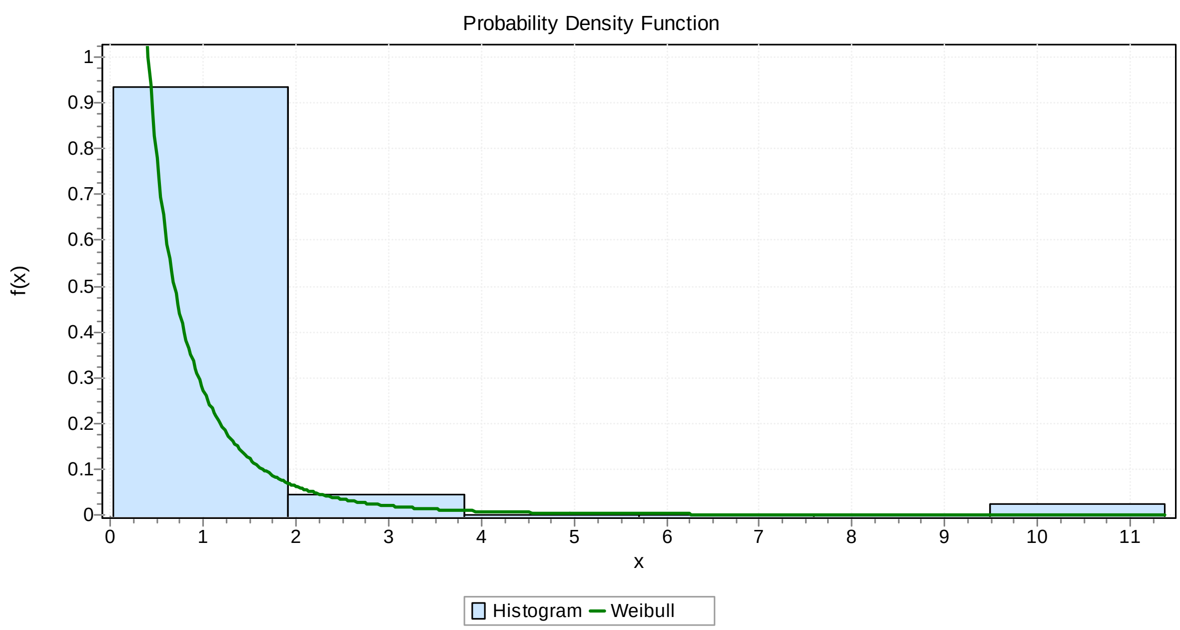


Table B.8 Goodness-of-fit and Distribution Parameters (Florida)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.23979	5	Fail to Reject H_0 at α=0.01	$\alpha_{1=}$ 0.12455 $\alpha_{2=}$ 1.01
Frechet	*** 0.2400	** 0.2002	* 0.1805	0.15215	4	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.40319 β=1.7103
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.116	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.29183 β=231.47
Gumbel	0.2400	0.2002	0.1805	0.32581	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=97.497 μ=11.274
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.1382	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.7166 μ=1.9869
Normal	0.2400	0.2002	0.1805	0.31542	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=125.04 μ=67.551
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.11142	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.43482 β=27.04

Figure B.8 Fitted Weibull distribution and histogram for Florida

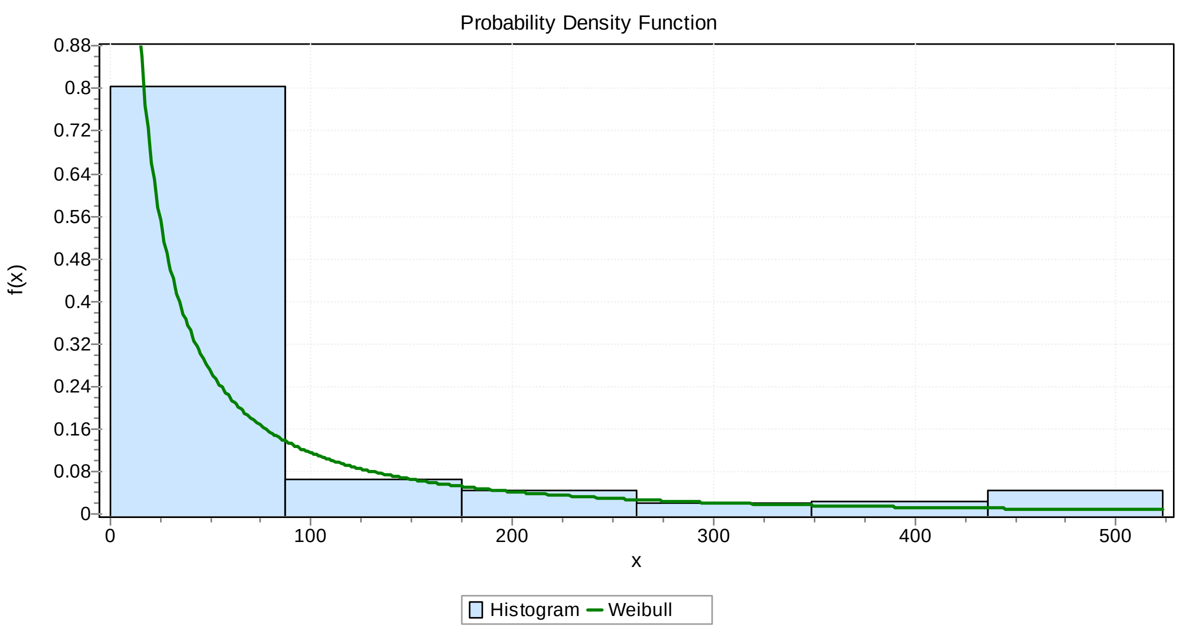


Table B.9 Goodness-of-fit and Distribution Parameters (Georgia)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.32698	5	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.12799$\alpha_{2=}$ 1.251
Frechet	*** 0.2400	** 0.2002	* 0.1805	0.17774	3	Fail to Reject H_0 at α=0.01,0.05& 0.1	α=0.46065 β=0.94093
Gamma	*** 0.2400	0.2002	0.1805	0.23202	4	Fail to Reject H_0 at α=0.01 and rejects H_0 at α=0.05& 0.1	α= 0.20146 β=139.03
Gumbel	0.2400	0.2002	0.1805	0.36869	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=48.655 μ=-0.07538
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.14339	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.4362 μ=1.2276
Normal	0.2400	0.2002	0.1805	0.34359	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=62.403 μ=28.009
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.10905	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.45931 β=11.242

Figure B.9 Fitted Weibull distribution and histogram for Georgia

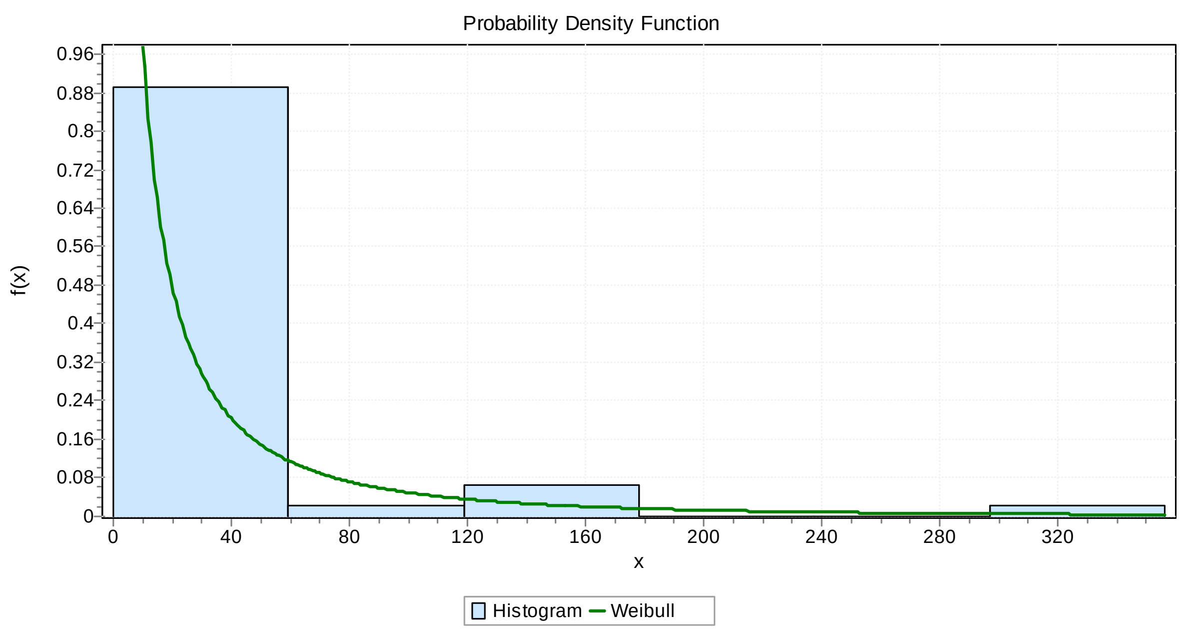


Table B.10 Goodness-of-fit and Distribution Parameters (Hawaii)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.93198	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_1=$ 0.02509 $\alpha_2=$ 0.65523
Frechet	0.2400	0.2002	0.1805	0.29954	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.44853 β=0.18867
Gamma	0.2400	0.2002	0.1805	0.72961	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.06276 β=279.3
Gumbel	0.2400	0.2002	0.1805	0.46173	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=54.553 μ=-13.961
Lognormal	0.2400	0.2002	0.1805	0.35831	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.512 μ=0.30331
Normal	0.2400	0.2002	0.1805	0.4961	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=69.967 μ=17.528
Weibull	0.2400	0.2002	0.1805	0.30371	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.37054 β=2.8071

Figure B.10 Fitted beta distribution and histogram for Hawaii

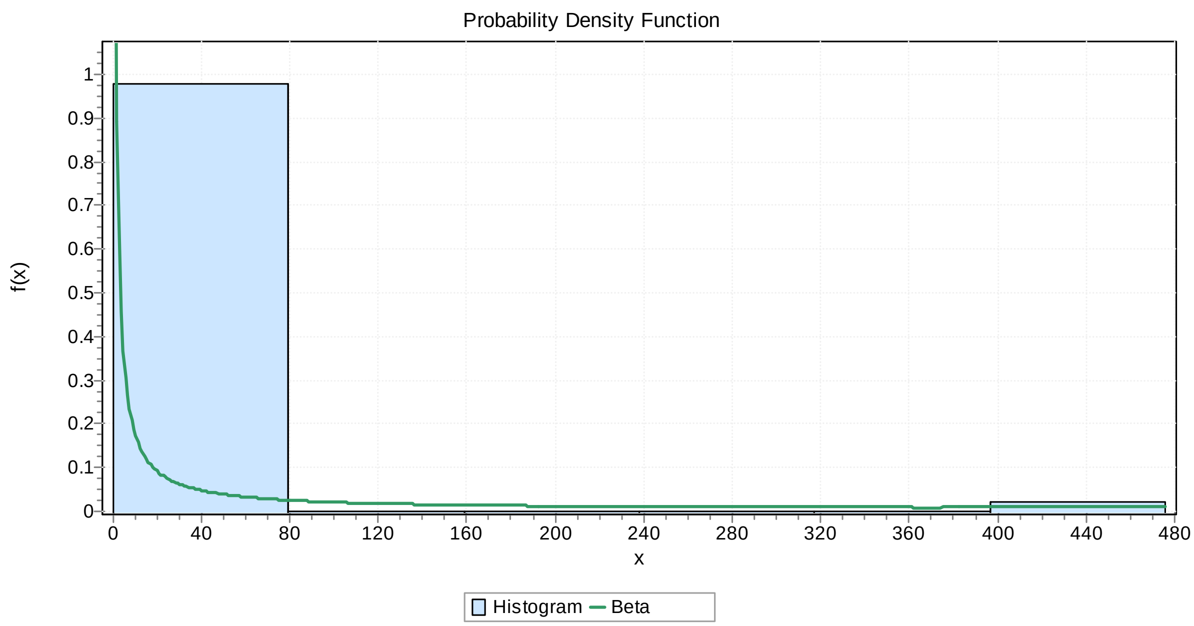


Table B.11 Goodness-of-fit and Distribution Parameters (Idaho)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.78304	7	Reject H_0 at α=0.01,0.05 & 0.1	α_1= 0.01625 α_2= 0.41534
Frechet	0.2400	0.2002	0.1805	0.1713	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.38104 β=0.42294
Gamma	0.2400	0.2002	0.1805	0.60413	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.04662 β=3091.2
Gumbel	0.2400	0.2002	0.1805	0.47685	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=520.39 μ=-156.27
Lognormal	*** 0.2400	** 0.2002	0.1805	0.18507	3	Fail to Reject H_0 at α=0.01,0.05 & Reject H_0 at α=0.1	σ=3.1032 μ=0.74413
Normal	0.2400	0.2002	0.1805	0.41455	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=667.42 μ=144.1
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.1742	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.30944 β=10.668

Figure B.11 Fitted Weibull distribution and histogram for Idaho

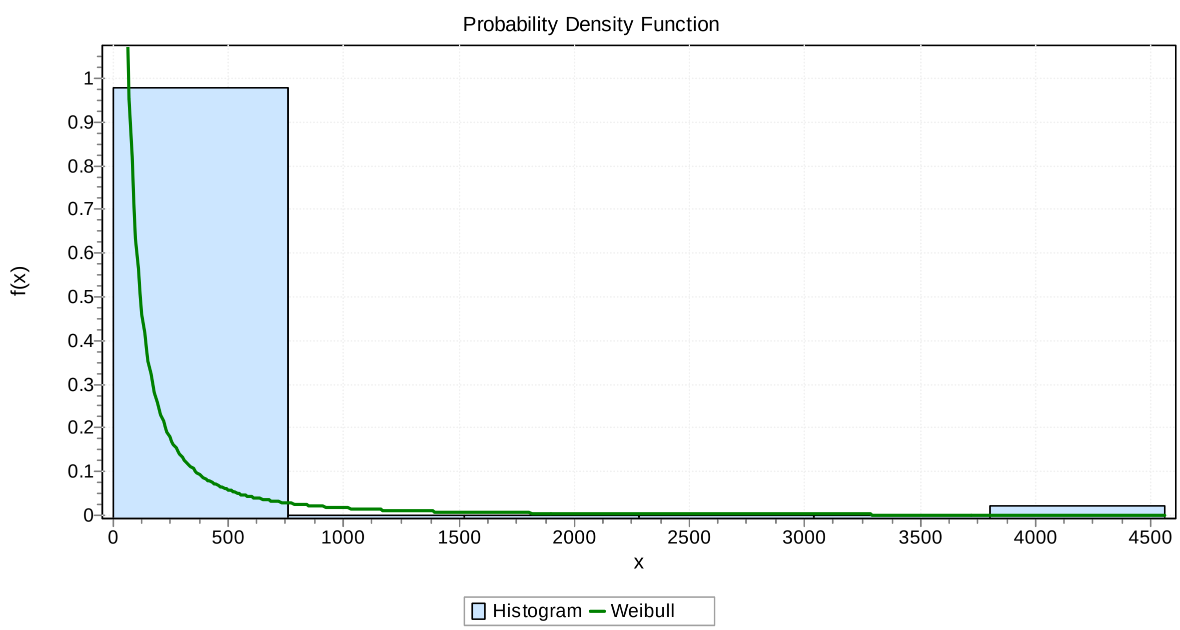


Table B.12 Goodness-of-fit and Distribution Parameters (Illinois)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.27088	5	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.11616$\alpha_{2=}$ 0.69897
Frechet	0.2400	0.2002	0.1805	0.16842	4	Fail to Reject H_0 at α=0.01,0.05& 0.1	α=0.386 β=6.8325
Gamma	*** 0.2400	** 0.2002	0.1805	0.16393	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.24882β=1604.7
Gumbel	0.2400	0.2002	0.1805	0.3981	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=624.12 μ=39.033
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.08997	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.8733 μ=3.4395
Normal	0.2400	0.2002	0.1805	0.33153	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=800.46 μ=399.28
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.1237	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.40103 β=124.77

Figure B.12 Fitted lognormal distribution and histogram for Illinois

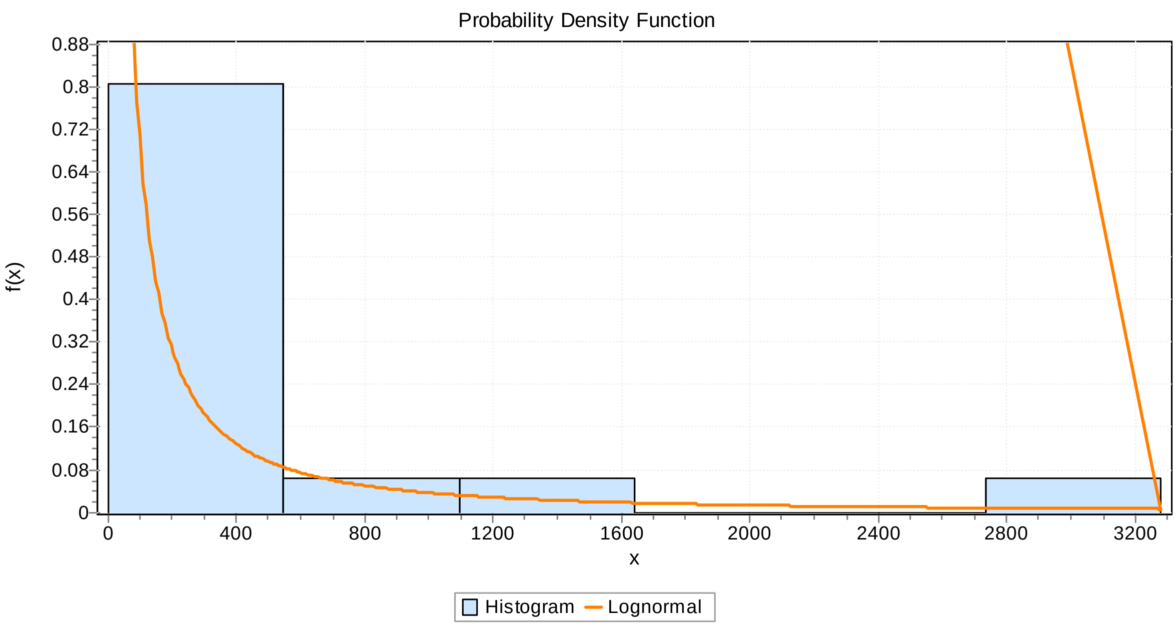


Table B.13 Goodness-of-fit and Distribution Parameters (Indiana)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.23939	5	Fail to Reject H_0 at α=0.01 and rejects H_0 at α=0.05& 0.1	$\alpha_{1=}$ 0.16043$\alpha_{2=}$ 1.3813
Frechet	0.2400	0.2002	0.1805	0.22144	4	Fail to Reject H_0 at α=0.01	α=0.41605 β=7.9147
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.13411	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.27262β=705.15
Gumbel	0.2400	0.2002	0.1805	0.3699	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=287.06 μ=26.536
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.15007	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5273 μ=3.4747
Normal	0.2400	0.2002	0.1805	0.30089	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=368.18 μ=192.23
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.08209	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.50973 β=101.87

Figure B.13 Fitted Weibull distribution and histogram for Indiana

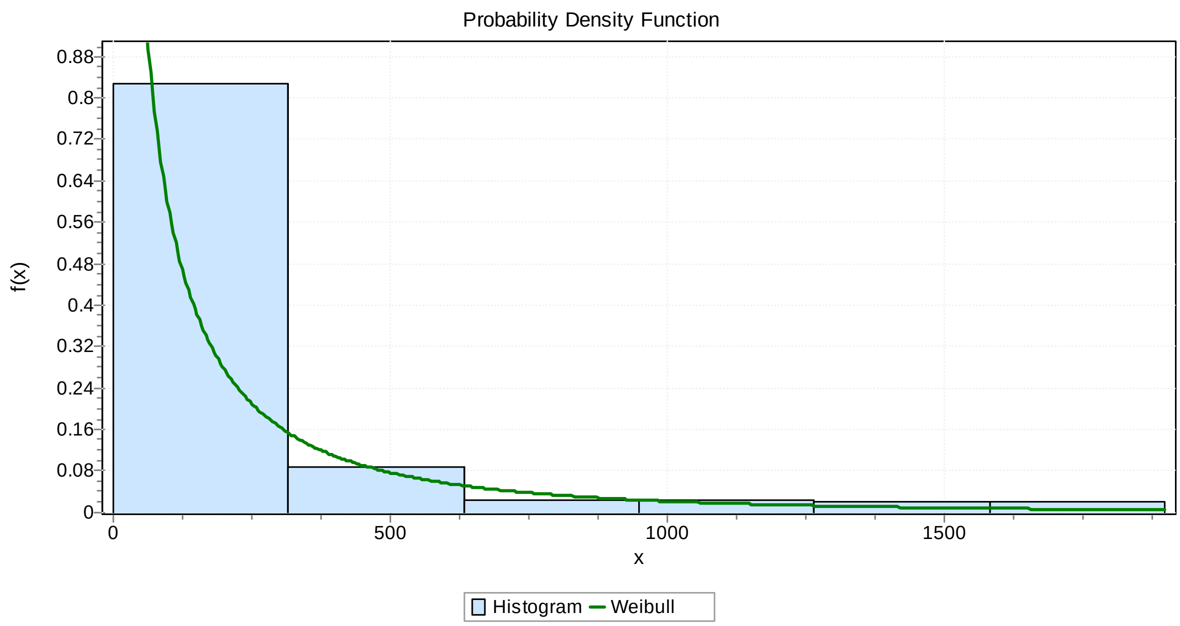


Table B.14 Goodness-of-fit and Distribution Parameters (Iowa)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.31125	4	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.08257$\alpha_{2=}$ 1.0724
Frechet	0.2400	0.2002	0.1805	0.14677	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.35177 β=3.2904
Gamma	0.2400	0.2002	0.1805	0.25717	3	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.13745 β=3129.6
Gumbel	0.2400	0.2002	0.1805	0.43678	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=904.67 μ=-92.023
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.11949	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.1979 μ=-2.8773
Normal	0.2400	0.2002	0.1805	0.36895	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1160.3 μ=430.16
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.8976	7	Reject H_0 at α=0.01,0.05 & 0.11	α=0.35043β=850.44

Figure B.14 Fitted Weibull distribution and histogram for Iowa

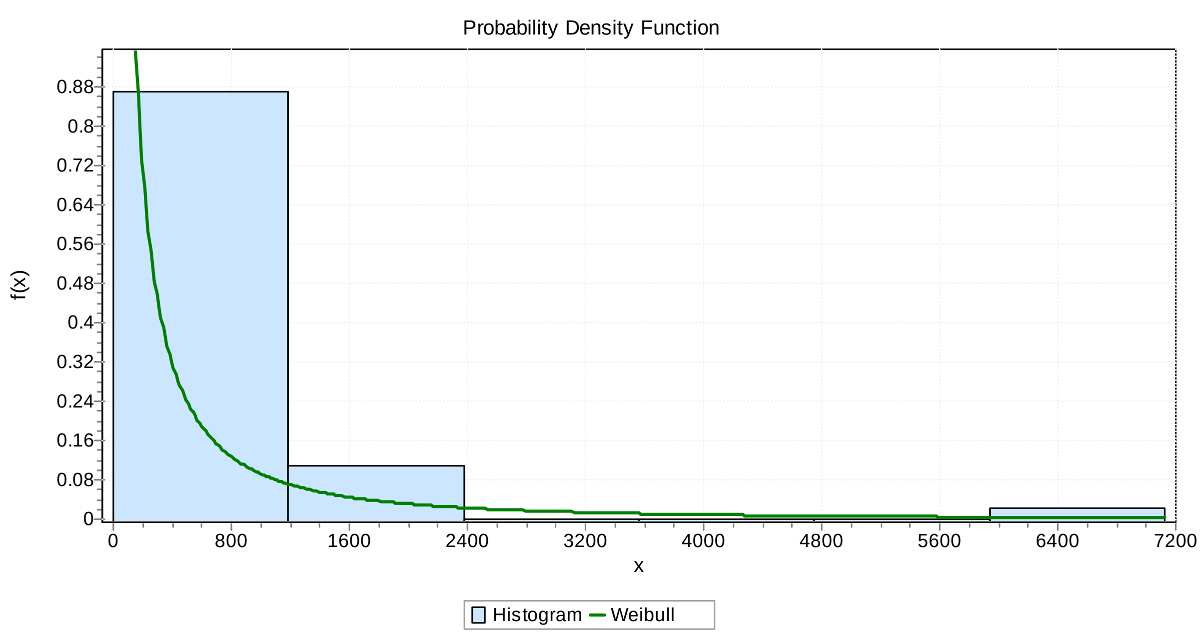


Table B.15 Goodness-of-fit and Distribution Parameters (Kansas)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.2392	5	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.0373 $\alpha_{2=}$ 0.69685
Frechet	0.2400	0.2002	0.1805	0.17833	4	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.38854 β=3.232
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.1051	2	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08564 β=5.967
Gumbel	0.2400	0.2002	0.1805	0.37189	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=152.34 μ=28.756
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.11553	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.0792 μ=-1.9796
Normal	0.2400	0.2002	0.1805	0.30649	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.7462 μ=0.51104
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.09396	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.54953 β=0.21479

Figure B.15 Fitted Weibull distribution and histogram for Kansas

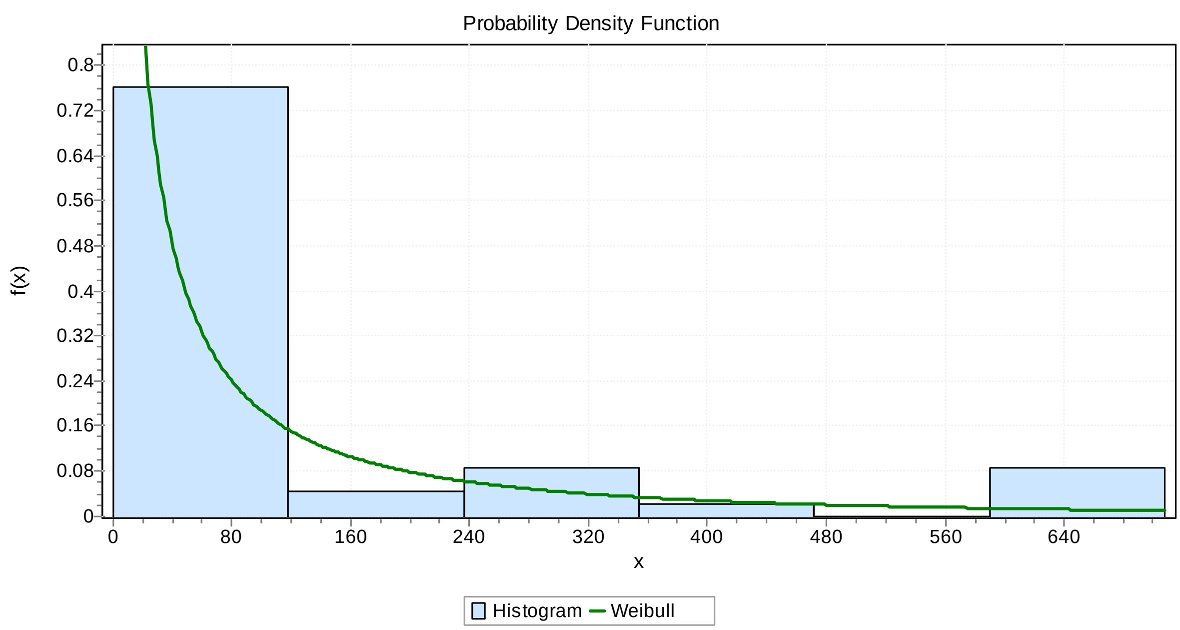


Table B.16 Goodness-of-fit and Distribution Parameters (Kentucky)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	** 0.2002	* 0.1805	0.10564	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α_{1}= 0.24354 α_{2}= 1.9092
Frechet	0.2400	0.2002	0.1805	0.181	4	Fail to Reject H_0 at α=0.01,0.05	α=0.30512 β=3.5932
Gamma	*** 0.2400	0.2002	0.1805	0.22777	5	Fail to Reject H_0 at α=0.01 & rejects H_0 at α=0.05& 0.1	α= 0.43403 β=728.97
Gumbel	0.2400	0.2002	0.1805	0.28512	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=374.45 μ=100.25
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.16737	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.4075 μ=3.1762
Normal	0.2400	0.2002	0.1805	0.25504	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=480.25 μ=316.39
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.13918	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.38 β=118.44

Figure B.16 Fitted beta distribution and histogram for Kentucky

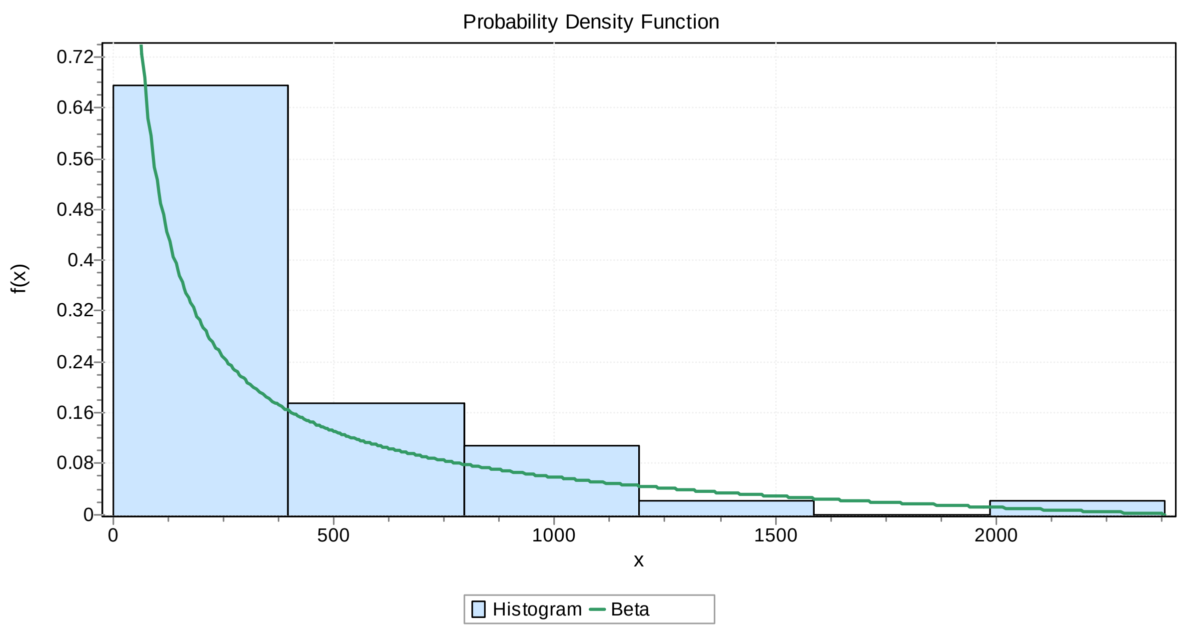


Table B.17 Goodness-of-fit and Distribution Parameters (Louisiana)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.22718	5	Fail to Reject H_0 at α=0.01	α_{1}= 0.07604 α_{2}= 0.79962
Frechet	0.2400	0.2002	0.1805	0.14115	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.31782 β=1.5043
Gamma	*** 0.2400	0.2002	0.1805	0.21139	4	Fail to Reject H_0 at α=0.01	α= 0.1989 β=2711.1
Gumbel	0.2400	0.2002	0.1805	0.47097	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=942.72 μ=-4.9287
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.14183	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.5803 μ=2.2663
Normal	0.2400	0.2002	0.1805	0.40742	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=1209.1 μ=539.22
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.14249	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.29314 β=59.477

Figure B.17 Fitted lognormal distribution and histogram for Louisiana

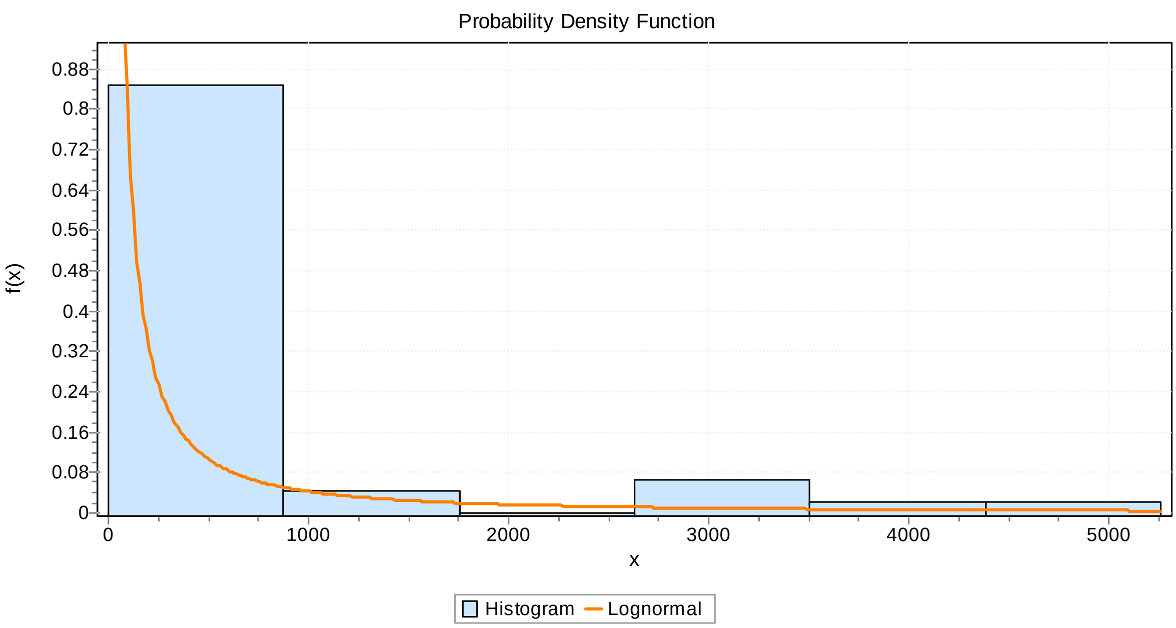


Table B.18 Goodness-of-fit and Distribution Parameters (Maine)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.3567	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.09158 $\alpha_{2=}$ 0.84141
Frechet	0.2400	0.2002	0.1805	0.29744	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.43424 β=0.21975
Gamma	0.2400	0.2002	0.1805	0.34556	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.17715 β=60.039
Gumbel	0.2400	0.2002	0.1805	0.36868	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=19.703 μ=-0.73692
Lognormal	0.2400	0.2002	0.1805	0.3495	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.4881 μ=0.14273
Normal	0.2400	0.2002	0.1805	0.3383	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=25.27 μ=10.636
Weibull	0.2400	0.2002	0.1805	0.32494	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.40767 β=3.1526

Figure B.18 Fitted Weibull distribution and histogram for Maine

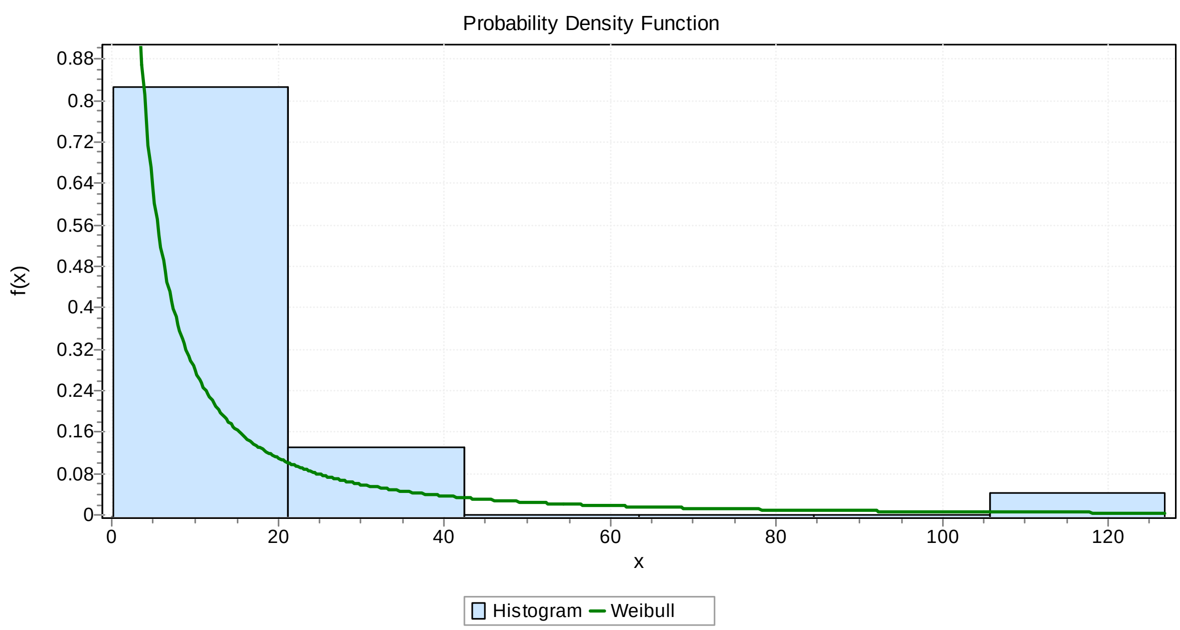


Table B.19 Goodness-of-fit and Distribution Parameters (Maryland)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.8209	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_1 = 0.01235$ $\alpha_2 = 0.033709$
Frechet	0.2400	0.2002	0.1805	0.21408	1	Fail to Reject H_0 at α=0.01	α=0.42037 β=0.20151
Gamma	0.2400	0.2002	0.1805	0.65064	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.04118 β=2018.2
Gumbel	0.2400	0.2002	0.1805	0.48729	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=319.33 μ=-101.21
Lognormal	0.2400	0.2002	0.1805	0.24905	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.8481 μ=-0.12169
Normal	0.2400	0.2002	0.1805	0.41963	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=409.56 μ=83.113
Weibull	*** 0.2400	0.2002	0.1805	0.22938	2	Fail to Reject H_0 at α=0.01	α=0.30436 β=4.1725

Figure B.19 Fitted Weibull distribution and histogram for Maryland

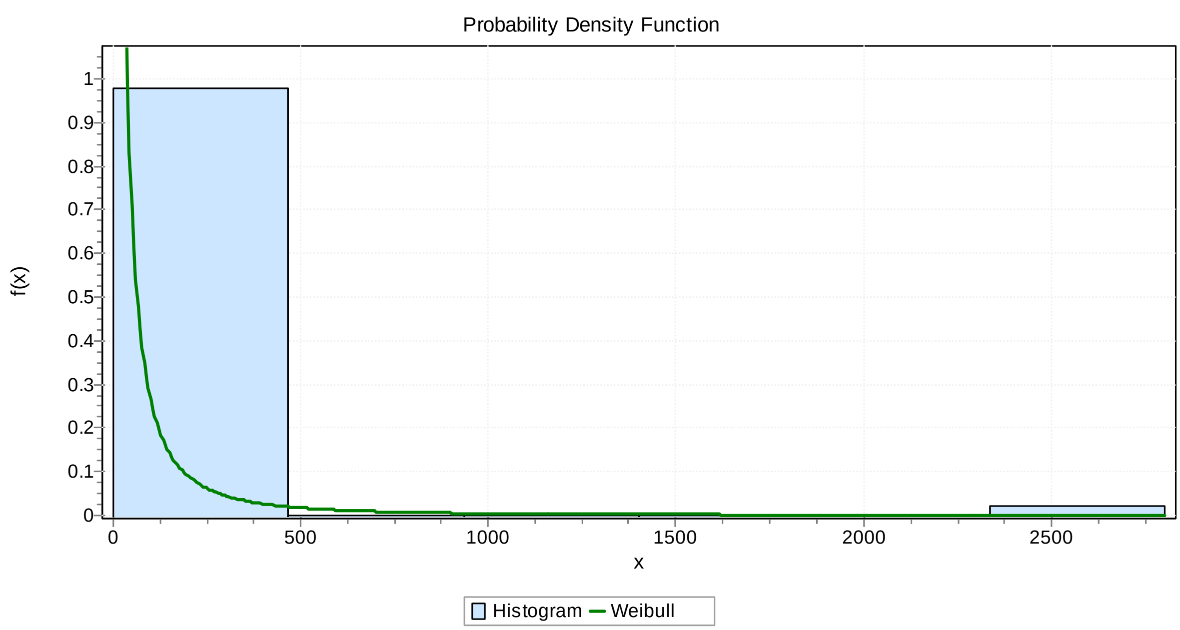


Table B.20 Goodness-of-fit and Distribution Parameters (Massachusetts)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.80925	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.01263 $\alpha_{2=}$ 0.33802
Frechet	0.2400	0.2002	0.1805	0.34944	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.37009 β=0.0975
Gamma	0.2400	0.2002	0.1805	0.65047	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.04203 β=3572.8
Gumbel	0.2400	0.2002	0.1805	0.50327	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=571.12 μ=-179.49
Lognormal	0.2400	0.2002	0.1805	0.36182	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=3.0012 μ=-0.64624
Normal	0.2400	0.2002	0.1805	0. 43259	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=732.48 μ=150.17
Weibull	0.2400	0.2002	0.1805	0.3552	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.2651 β=2.8652

Figure B.20 Fitted Weibull distribution and histogram for Massachusetts

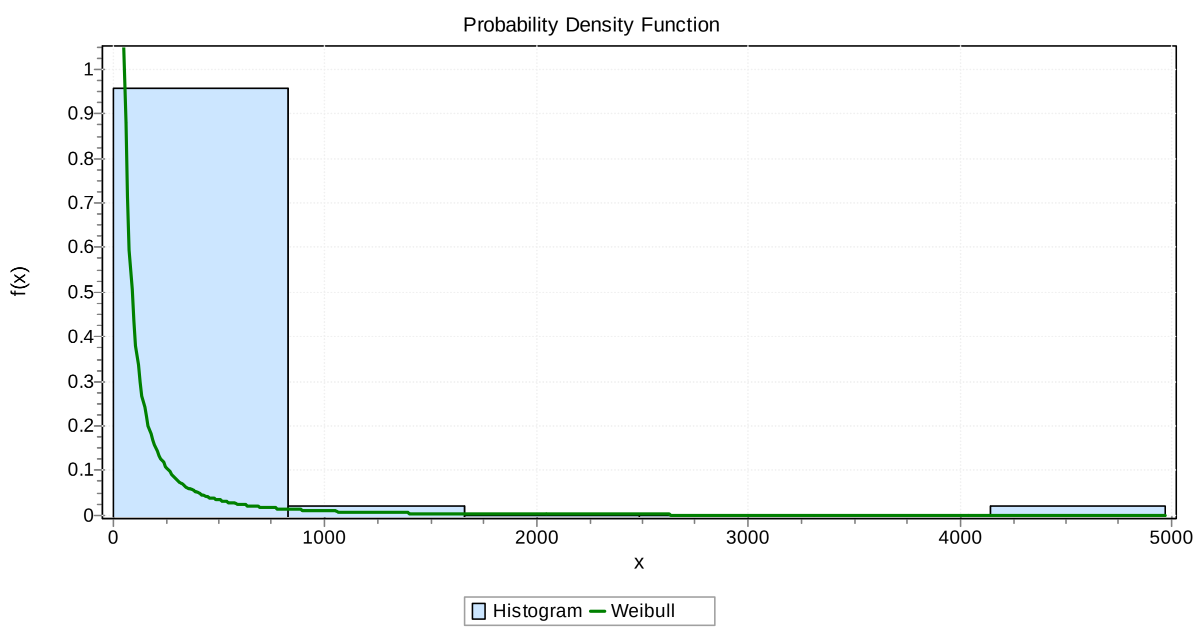


Table B.21 Goodness-of-fit and Distribution Parameters (Michigan)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.78328	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.01852 $\alpha_{2=}$ 0.38736
Frechet	0.2400	0.2002	0.1805	0.232	1	Fail to Reject H_0 at α=0.01	α=0.45764 β=0.19928
Gamma	0.2400	0.2002	0.1805	0.60549	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.05598 β=705.04
Gumbel	0.2400	0.2002	0.1805	0.50963	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=130.07 μ=-35.606
Lognormal	0.2400	0.2002	0.1805	0.26067	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.6011 μ=0.2576
Normal	0.2400	0.2002	0.1805	0.43914	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=166.82 μ=39.471
Weibull	0.2400	0.2002	0.1805	0.24835	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.33444 β=3.1638

Figure B.21 Fitted Weibull distribution and histogram for Michigan

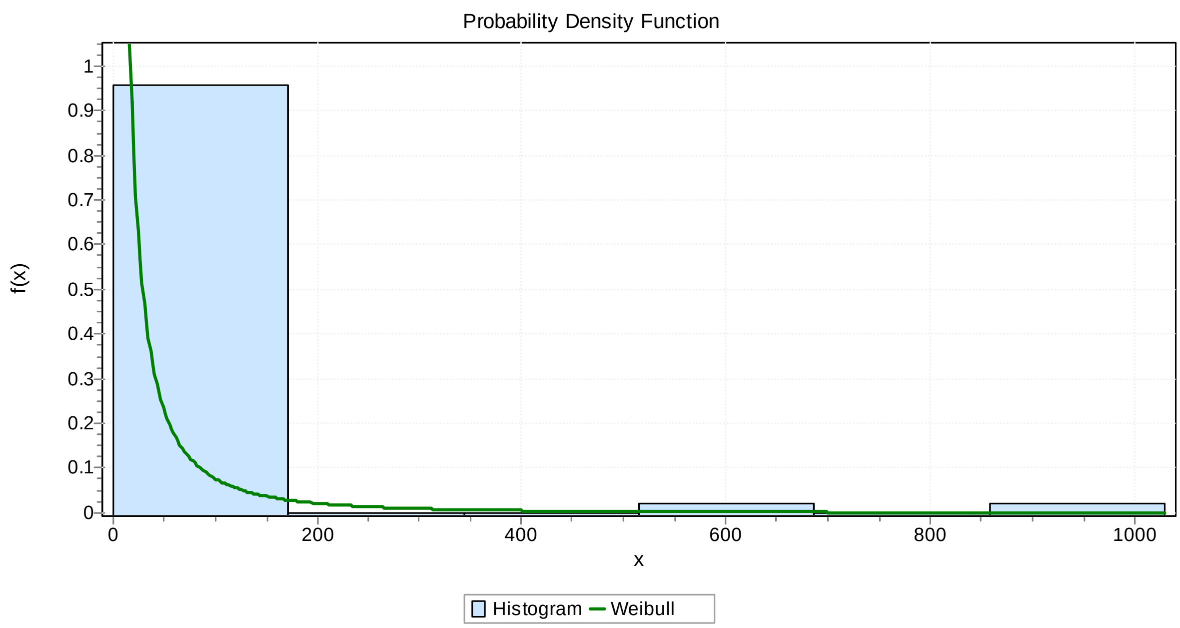


Table B.22 Goodness-of-fit and Distribution Parameters (Minnesota)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.23964	5	Fail to Reject H_0 at α=0.01	$\alpha_{1=}$ 0.09965 $\alpha_{2=}$ 1.2048
Frechet	0.2400	0.2002	0.1805	0.09466	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36629 β=1.4305
Gamma	*** 0.2400	0.2002	0.1805	0.19464	4	Fail to Reject H_0 at α=0.01	α= 0.209 β=951.32
Gumbel	0.2400	0.2002	0.1805	0.42504	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=339.1 μ=3.0954
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.10779	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.1297 μ=1.9772
Normal	0.2400	0.2002	0.1805	0.36003	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=434.91 μ=198.83
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.15198	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.33466 β=35.613

Figure B.22 Fitted lognormal distribution and histogram for Minnesota

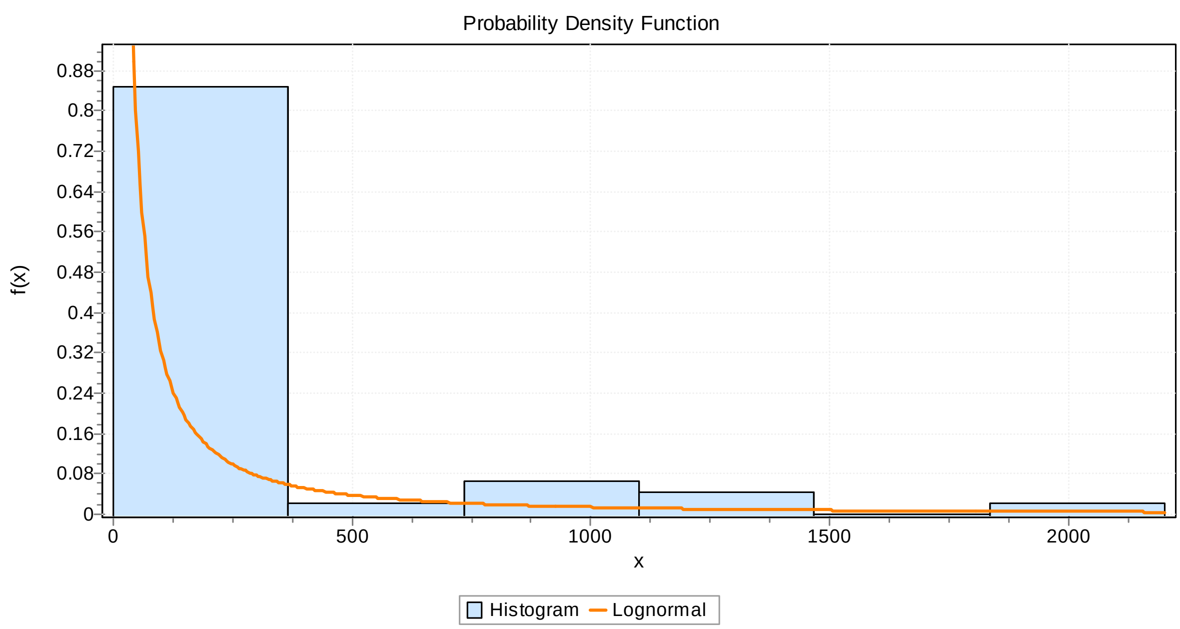


Table B.23 Goodness-of-fit and Distribution Parameters (Mississippi)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.44275	7	Reject H_0 at α=0.01,0.05 & 0.1	α_1= 0.05343 α_2= 0.68208
Frechet	0.2400	0.2002	0.1805	0.174	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.40729 β=2.5707
Gamma	0.2400	0.2002	0.1805	0.3599	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.11286 β=1395.8
Gumbel	0.2400	0.2002	0.1805	0.44213	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=365.62 μ=-53.507
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.10487	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.7597 μ=2.4064
Normal	0.2400	0.2002	0.1805	0.37145	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=468.92 μ=157.53
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.08254	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.40525 β=42.407

Figure B.23 Fitted Weibull distribution and histogram for Mississippi

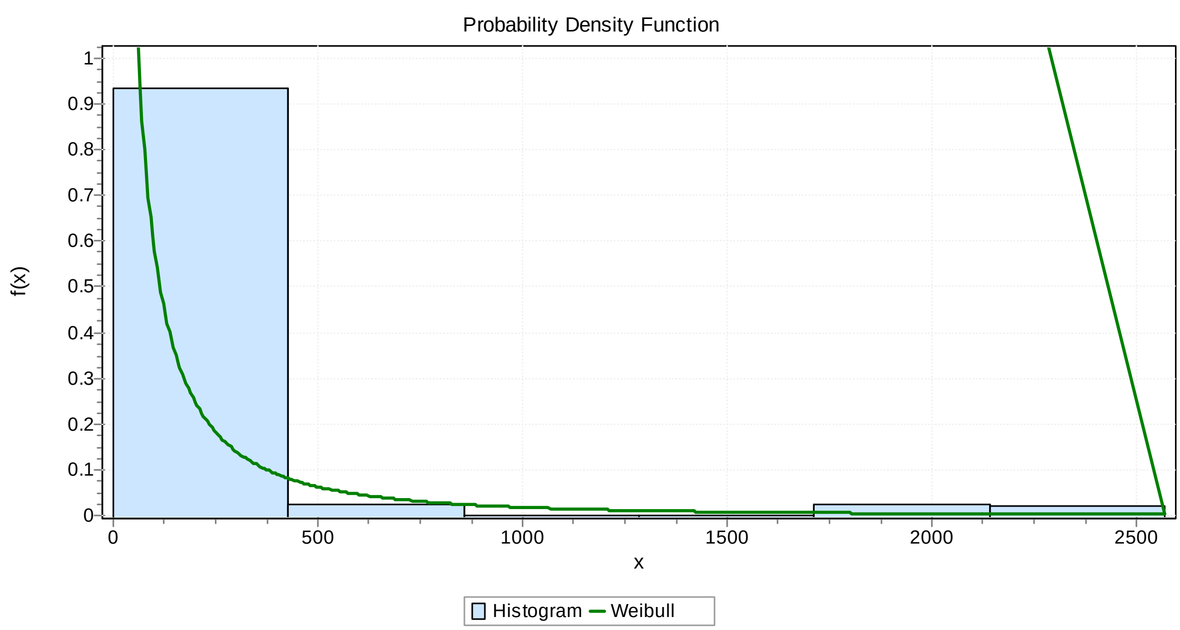


Table B.24 Goodness-of-fit and Distribution Parameters (Missouri)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.25775	5	Reject H_0 at α=0.01,0.05 & 0.1	α_{1}= 0.11516 α_{2}= 1.111
Frechet	0.2400	0.2002	0.1805	0.1343	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.3725 β=5.1655
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.16141	4	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.19485 β=1786.2
Gumbel	0.2400	0.2002	0.1805	0.37584	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=614.75 μ=-6.8102
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.07617	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.9768 μ=3.2223
Normal	0.2400	0.2002	0.1805	0.3295	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=788.45 μ=348.03
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.06265	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.3936 β=104.97

Figure B.24 Fitted Weibull distribution and histogram for Missouri

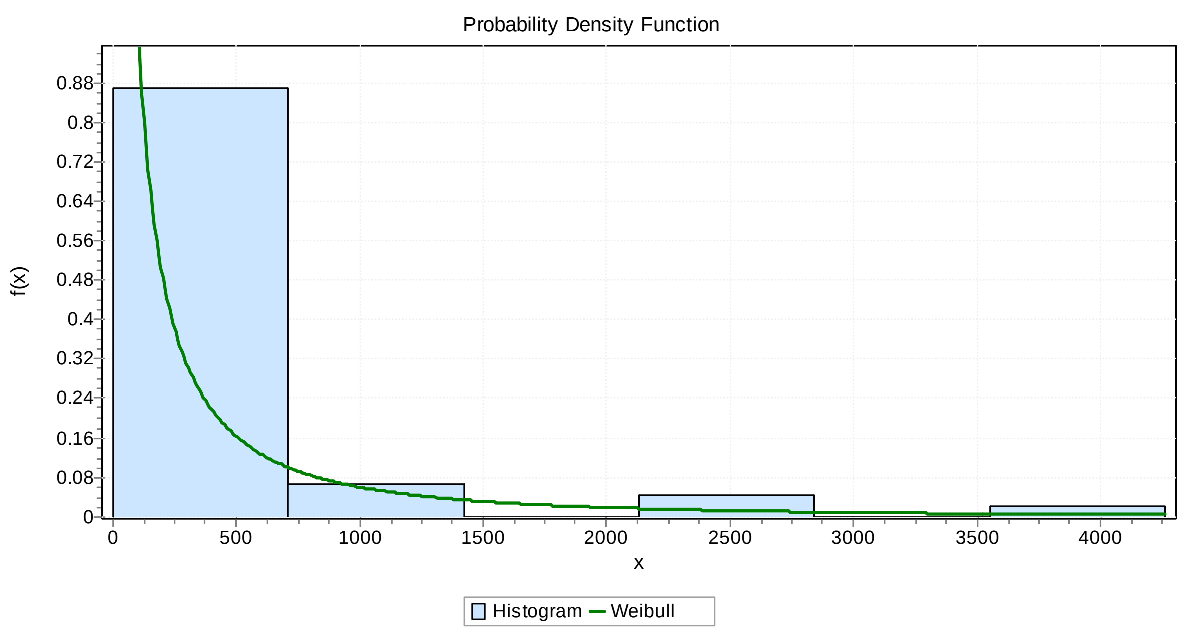


Table B.25 Goodness-of-fit and Distribution Parameters (Montana)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.6322	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.03579 $\alpha_{2=}$ 0.53199
Frechet	0.2400	0.2002	0.1805	0.17087	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.43623 β=0.41609
Gamma	0.2400	0.2002	0.1805	0.43379	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08763 β=769.57
Gumbel	0.2400	0.2002	0.1805	0.50681	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=177.62 μ=-35.091
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.17494	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.7399 μ=0.52298
Normal	0.2400	0.2002	0.1805	0.43945	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=227.81 μ=67.434
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.16623	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.33919 β=7.1136

Figure B.25 Fitted Weibull distribution and histogram for Montana

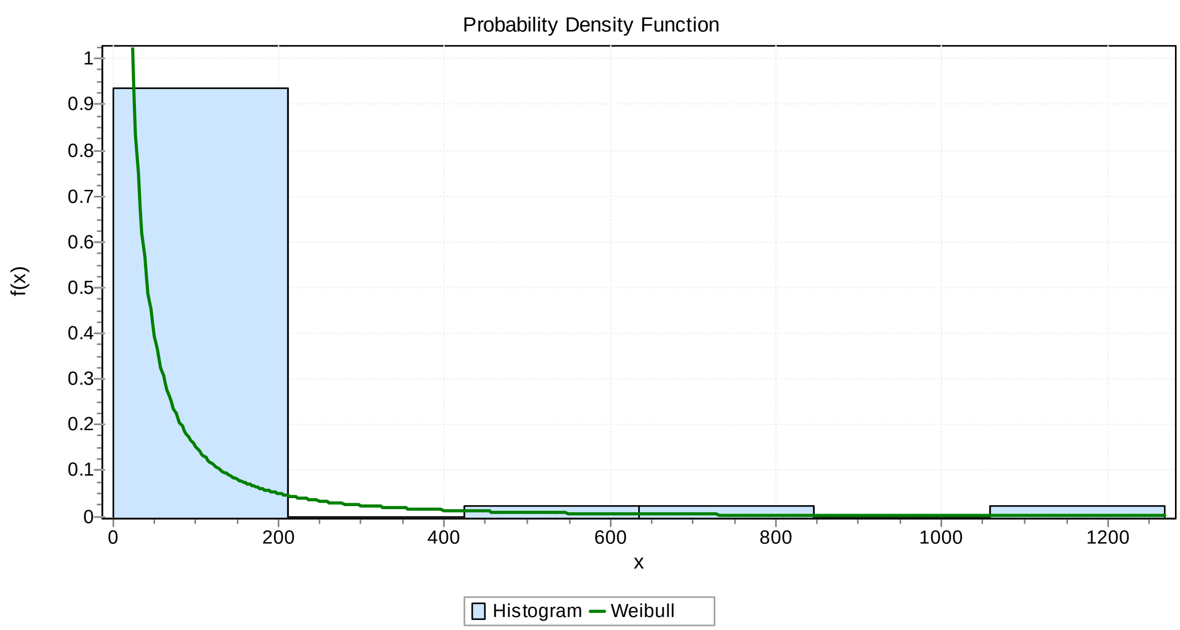


Table B.26 Goodness-of-fit and Distribution Parameters (Nebraska)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.20668	5	Fail to Reject H_0 at α=0.01,0.05& Reject H_0 at α=0.1	$\alpha_{1=}$ 0.14938 $\alpha_{2=}$ 1.6631
Frechet	0.2400	0.2002	0.1805	0.20331	4	Fail to Reject H_0 at α=0.01	α=0.36531 β=2.523
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.1134	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.27375 β=414.32
Gumbel	0.2400	0.2002	0.1805	0.36759	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=169.02 μ=15.86
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.14525	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.893 μ=2.5221
Normal	0.2400	0.2002	0.1805	0.30057	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=216.78 μ=113.42
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.12837	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.43647 β=48.3

Figure B.26 Fitted gamma distribution and histogram for Nebraska

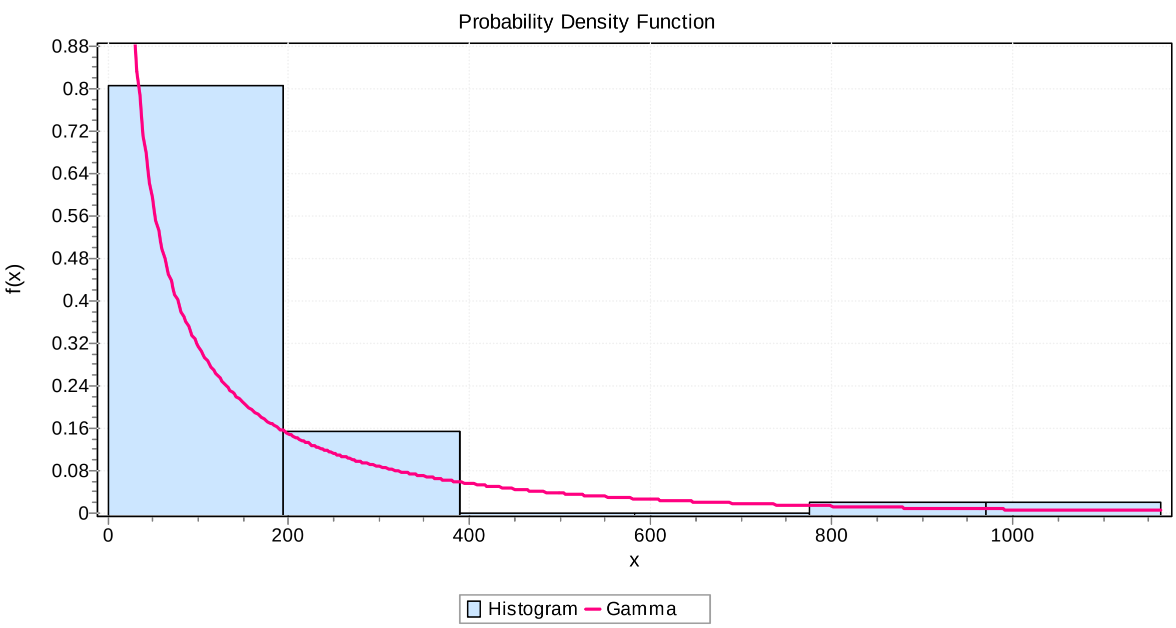


Table B.27 Goodness-of-fit and Distribution Parameters (Nevada)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.63669	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.0349$\alpha_{2=}$ 0.50257
Frechet	0.2400	0.2002	0.1805	0.2104	1	Fail to Reject H_0 at α=0.01	α=0.39933 β=0.14745
Gamma	0.2400	0.2002	0.1805	0.40905	5	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08865 β=407.82
Gumbel	0.2400	0.2002	0.1805	0.47939	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=94.673 μ=-18.494
Lognormal	0.2400	0.2002	0.1805	0.26276	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.9473 μ=0.39479
Normal	0.2400	0.2002	0.1805	0.40894	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=121.42 μ=36.152
Weibull	*** 0.2400	0.2002	0.1805	0.23768	2	Fail to Reject H_0 at α=0.01	α=0.31628 β=3.2109

Figure B.27 Fitted Weibull distribution and histogram for Nevada

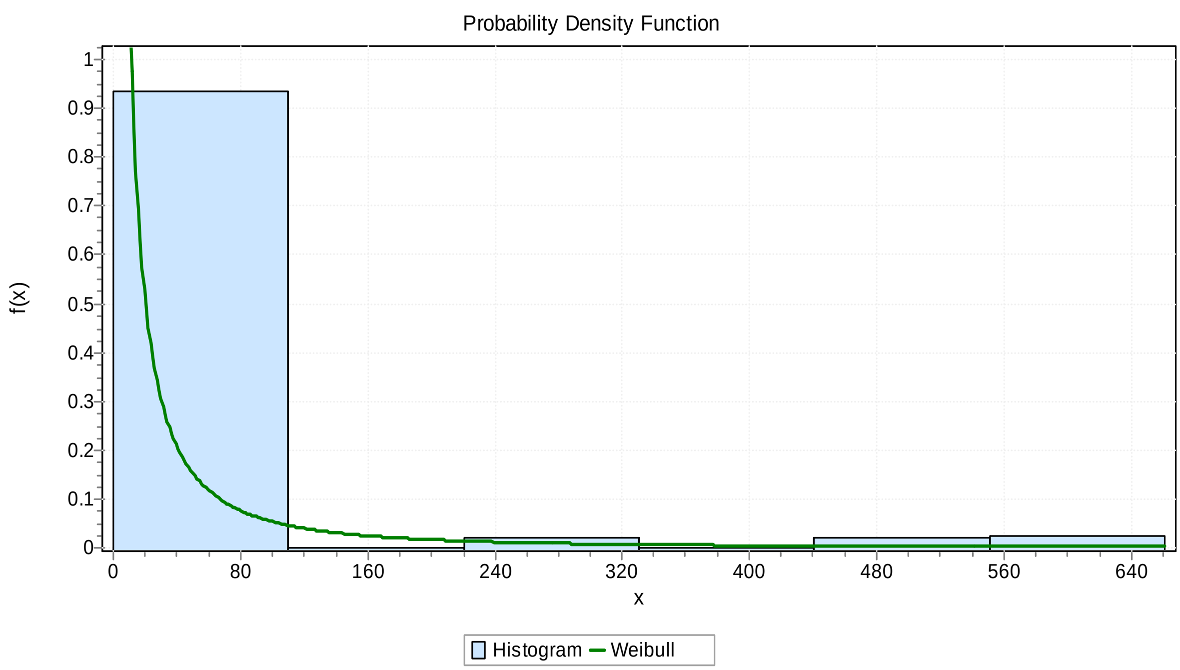


Table B.28 Goodness-of-fit and Distribution Parameters (New Hampshire)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.30577	1	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.00735 $\alpha_{2=}$ 0.25716
Frechet	0.2400	0.2002	0.1805	0.31686	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.50323 β=0.13186
Gamma	0.2400	0.2002	0.1805	0.7371	7	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.03555 β=921.78
Gumbel	0.2400	0.2002	0.1805	0.54657	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=135.51 μ=-45.449
Lognormal	0.2400	0.2002	0.1805	0.34092	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.3679 μ=0.76482
Normal	0.2400	0.2002	0.1805	0.47618	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=173.8 μ=32.768
Weibull	0.2400	0.2002	0.1805	0.31891	3	Reject H_0 at α=0.01,0.05 & 0.1	α=0.33445 β=1.7484

Figure B.28 Fitted gamma distribution and histogram for New Hampshire

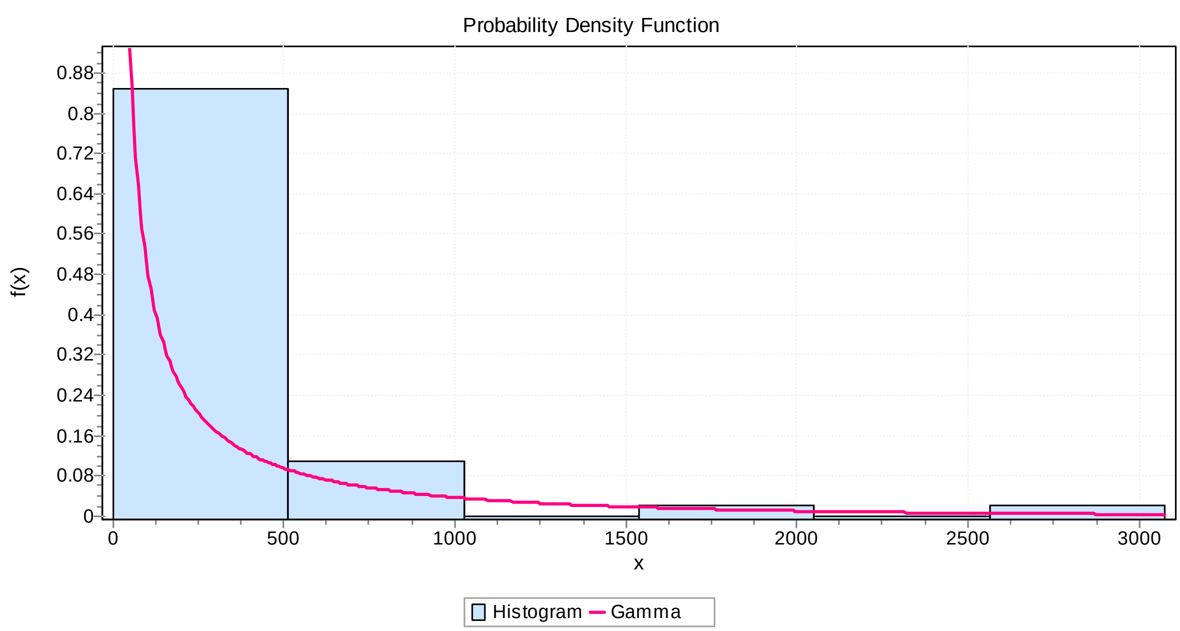


Table B.29 Goodness-of-fit and Distribution Parameters (New Jersey)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.40798	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.07887 $\alpha_{2=}$ 0.91462
Frechet	0.2400	0.2002	0.1805	0.26584	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.28967 β=0.23803
Gamma	0.2400	0.2002	0.1805	0.269	2	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.14232 β=1449.4
Gumbel	0.2400	0.2002	0.1805	0.44467	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=426.34 μ=-39.809
Lognormal	0.2400	0.2002	0.1805	0.32073	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=3.8085 μ=0.61775
Normal	0.2400	0.2002	0.1805	0.38038	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=546.8 μ=206.28
Weibull	0.2400	0.2002	0.1805	0.30074	3	Reject H_0 at α=0.01,0.05 & 0.1	α=0.26186 β=13.55

Figure B.29 Fitted gamma distribution and histogram for New Jersey

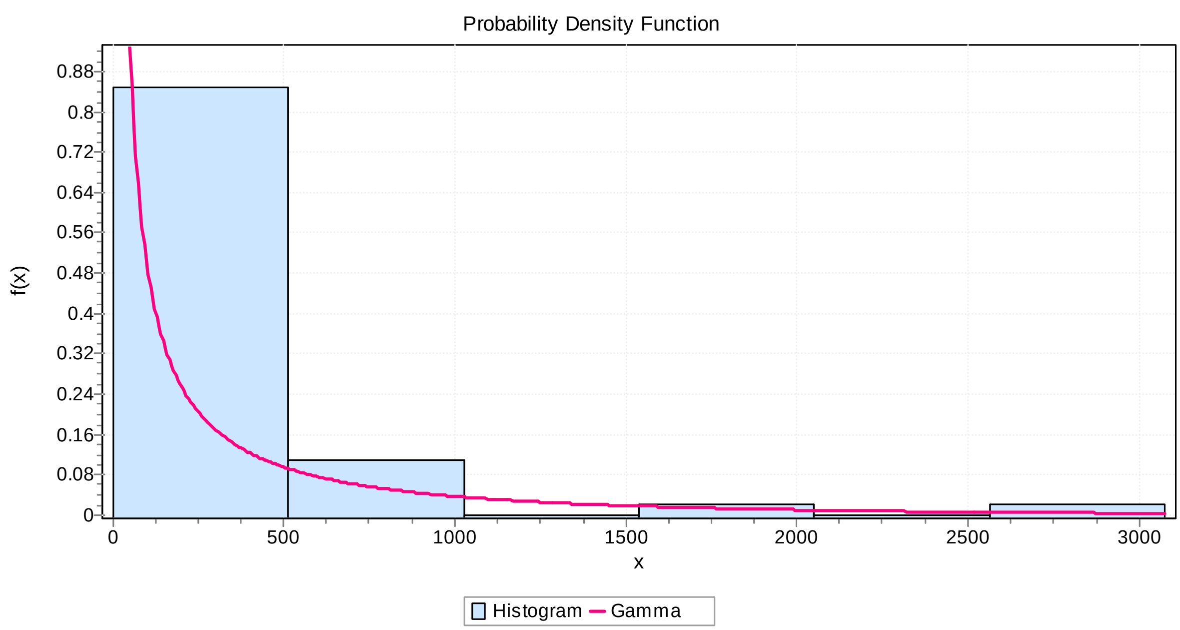


Table B.30 Goodness-of-fit and Distribution Parameters (New Mexico)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.74013	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.02345 $\alpha_{2=}$ 0.54466
Frechet	0.2400	0.2002	0.1805	0.22736	2	Fail to Reject H_0 at α=0.01	α=0.43397 β=0.24648
Gamma	0.2400	0.2002	0.1805	0.59401	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.0562 β=509.13
Gumbel	0.2400	0.2002	0.1805	0.42137	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=94.108 μ=-25.707
Lognormal	0.2400	0.2002	0.1805	0.26059	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.6848 μ=0.00923
Normal	0.2400 ***	0.2002	0.1805	0.40636	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=120.7 μ=28.614
Weibull	0.2400	0.2002	0.1805	0.22336	1	Fail to Reject H_0 at α=0.01& Reject H_0 α=at 0.05& 0.1	α=0.35888 β=4.1009

Figure B.30 Fitted Weibull distribution and histogram for New Mexico

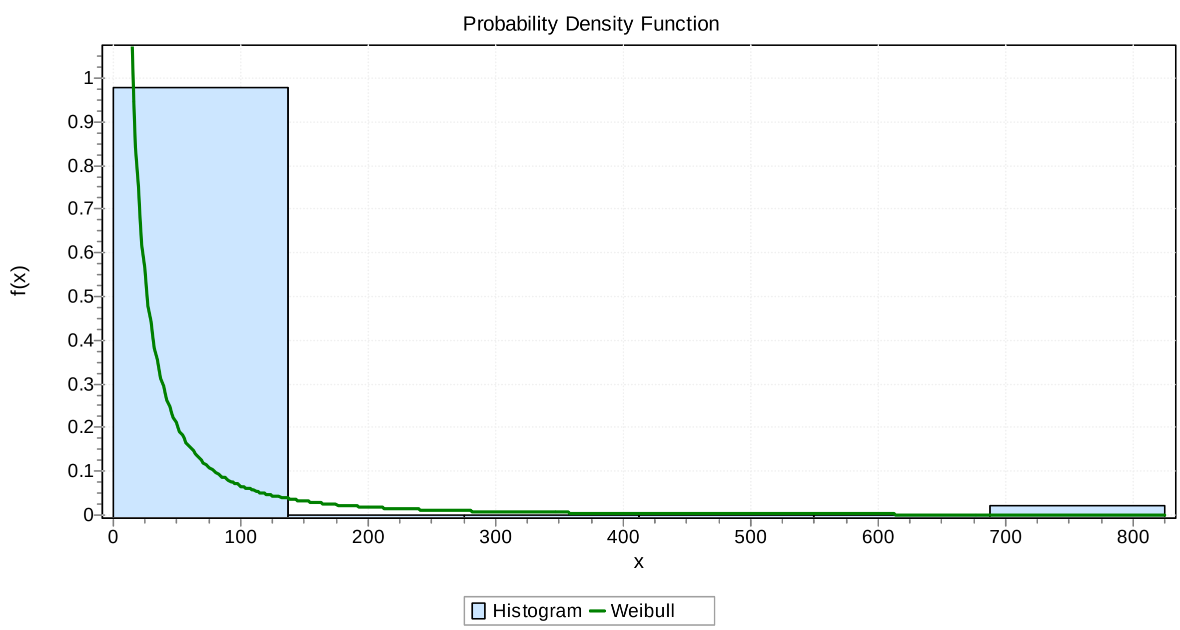


Table B.31 Goodness-of-fit and Distribution Parameters (New York)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.56329	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.02134 $\alpha_{2=}$ 0.51224
Frechet	0.2400	0.2002	0.1805	0.23395	3	Fail to Reject H_0 at α=0.01	α=0.34778 β=2.3495
Gamma	0.2400	0.2002	0.1805	0.57333	7	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.05315 β=5996.3
Gumbel	0.2400	0.2002	0.1805	0.45551	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1077.8 μ=-303.45
	***	**	*				
Lognormal	0.2400	0.2002	0.1805	0.1503	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.1192 μ=2.5743
Normal	0.2400	0.2002	0.1805	0.40887	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=1382.4 μ=318.69
	***	**	*				
Weibull	0.2400	0.2002	0.1805	0.12646	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36976 β=58.34

Figure B.31 Fitted Weibull distribution and histogram for New York

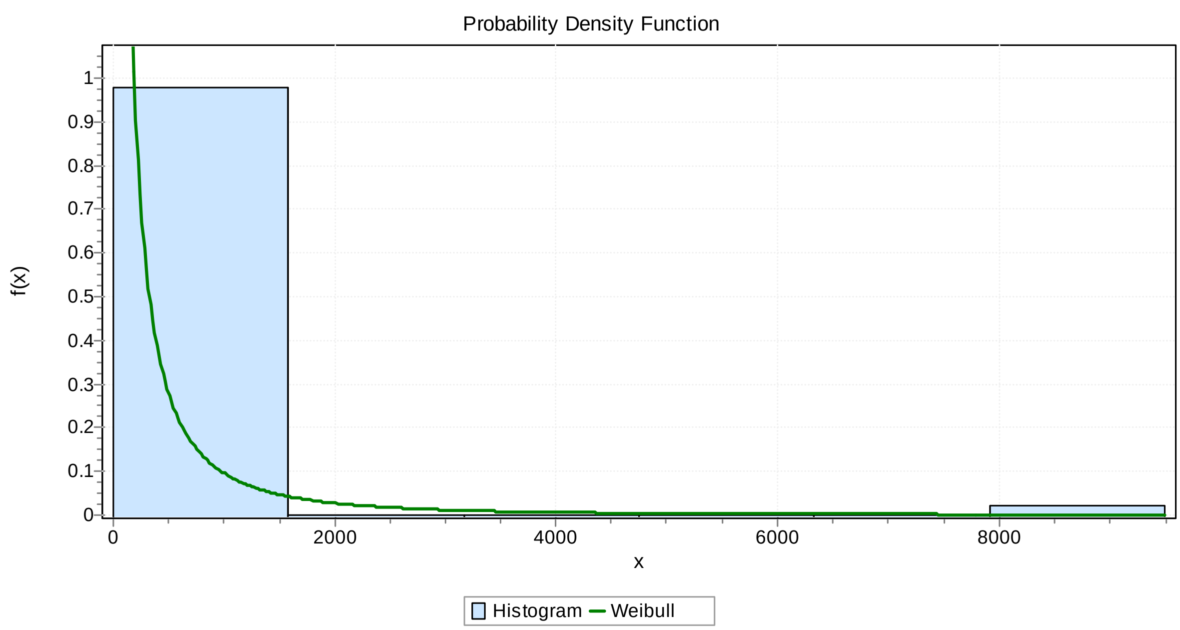


Table B.32 Goodness-of-fit and Distribution Parameters (North Carolina)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.5027	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.03147 $\alpha_{2=}$ 0.46144
Frechet	0.2400	0.2002	0.1805	0.19511	3	Fail to Reject H_0 at α=0.01,0.05	α=0.39007 β=1.4945
Gamma	0.2400	0.2002	0.1805	0.4528	5	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08472 β=1919.8
Gumbel	0.2400	0.2002	0.1805	0.48675	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=435.68 μ=-88.841
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.165	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.8891 μ=1.9369
Normal	0.2400	0.2002	0.1805	0.41655	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=558.78 μ=162.64
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.1212	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.37056 β=28.82

Figure B.32 Fitted Weibull distribution and histogram for North Carolina

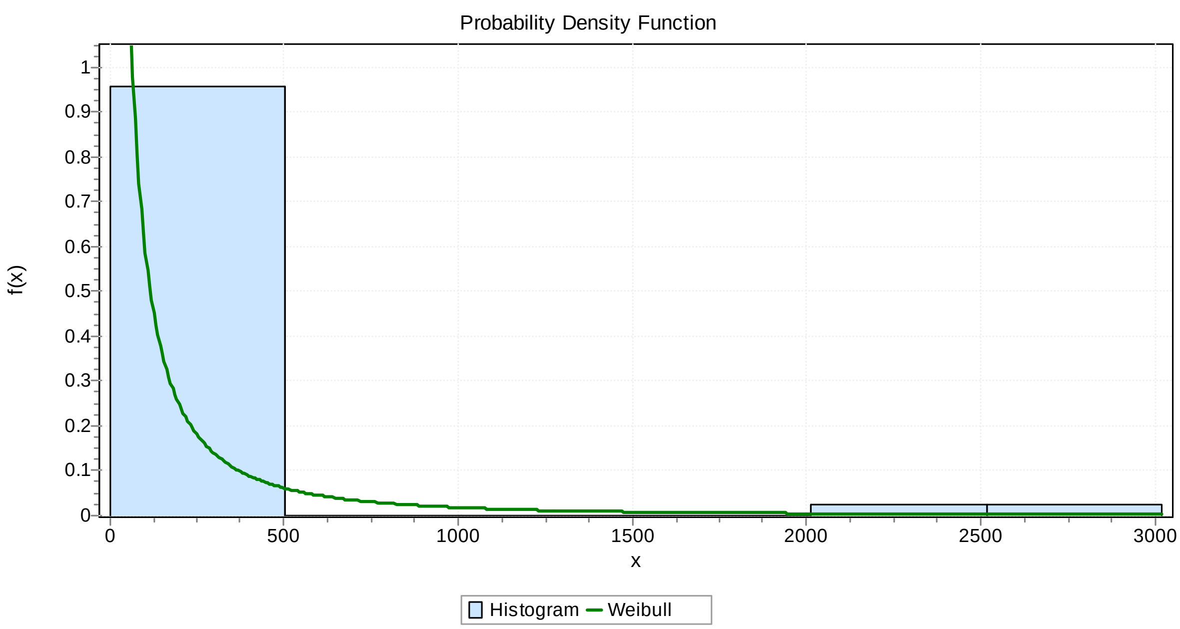


Table B.33 Goodness-of-fit and Distribution Parameters (North Dakota)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.65368	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.03153 $\alpha_{2=}$ 0.62075
Frechet	0.2400	0.2002	0.1805	0.21701	1	Fail to Reject H_0 at α=0.01	α=0.33741 β=0.35109
Gamma	0.2400	0.2002	0.1805	0.46854	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.06972 β=2049.4
Gumbel	0.2400	0.2002	0.1805	0.45886	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=421.94 μ=-100.66
Lognormal	0.2400	0.2002	0.1805	0.24967	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=3.3834 μ=0.73847
Normal	0.2400	0.2002	0.1805	0.39588	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=541.15 μ=142.89
Weibull	*** 0.2400	0.2002	0.1805	0.21975	2	Fail to Reject H_0 at α=0.01& Reject H_0 at α=0.05&0.1	α=0.29678 β=12.057

Figure B.33 Fitted Weibull distribution and histogram for North Dakota

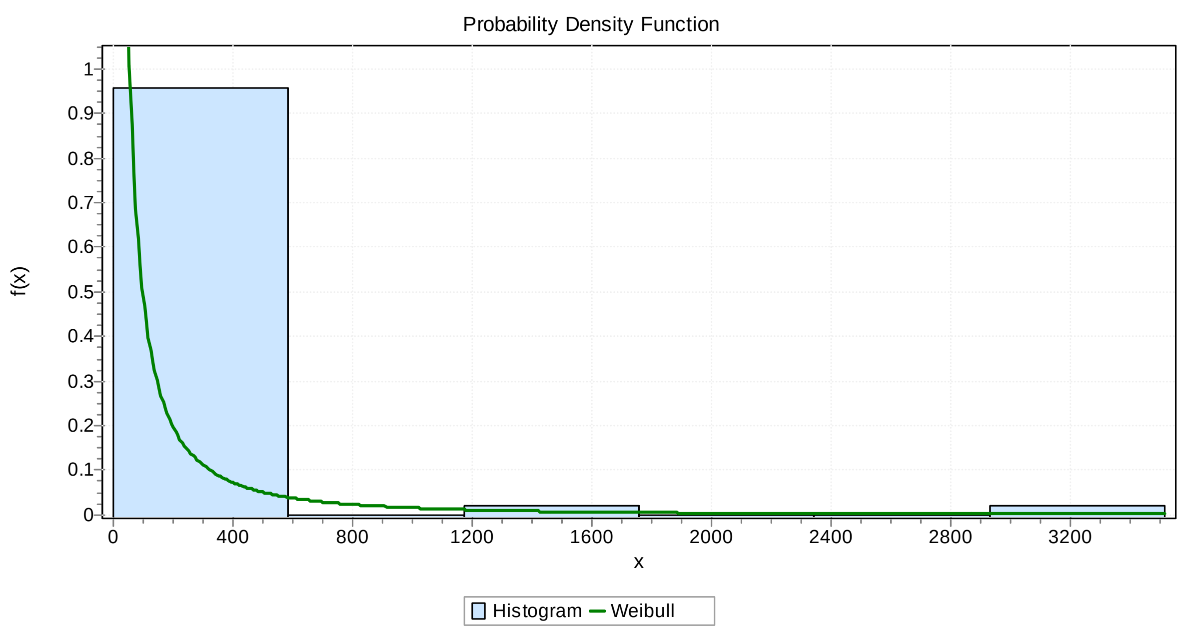


Table B.34 Goodness-of-fit and Distribution Parameters (Ohio)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.33175	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.10483 $\alpha_{2=}$0.85078
Frechet	0.2400	0.2002	0.1805	0.25262	4	Reject H_0 at α=0.01,0.05 & 0.1	α=0.36124 β=3.1683
Gamma	*** 0.2400	0.2002	0.1805	0.20726	3	Fail to Reject H_0 at α=0.01& Reject H_0 at α=0.05& 0.1	α= 0.19903 β=708.47
Gumbel	0.2400	0.2002	0.1805	0.38791	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=246.43 μ=-1.2424
Lognormal	*** 0.2400	** 0.2002	0.1805	0.1864	2	Fail to Reject H_0 at α=0.01 & 0.05& Reject H_0 at α=0.1	σ=2.8786 μ=2.7699
Normal	0.2400	0.2002	0.1805	0.32776	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=316.06 μ=141.0
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.11525	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.45089 β=58.594

Figure B.34 Fitted Weibull distribution and histogram for Ohio

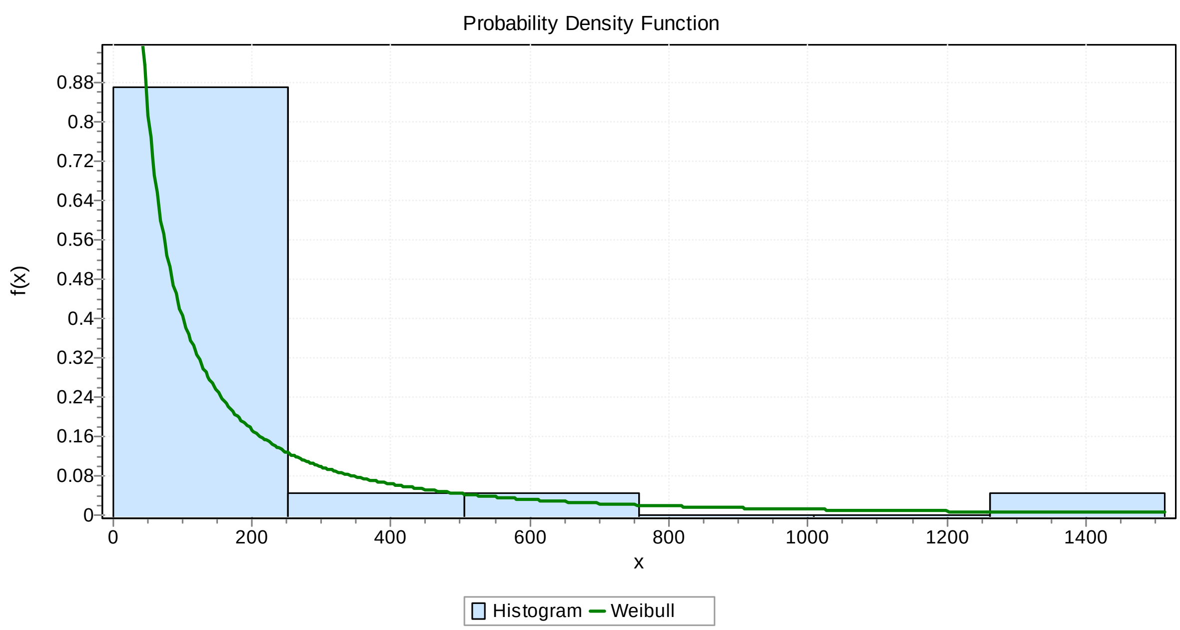


Table B.35 Goodness-of-fit and Distribution Parameters (Oklahoma)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	0.2002	0.1805	0.22327	5	Fail to Reject H_0 at α=0.01&Reject H_0 at α=0.05 & 0.1	$\alpha_{1=}$ 0.13583 $\alpha_{2=}$ 1.2035
Frechet	0.2400	0.2002	0.1805	0.15557	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.34661 β=1.453
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.14455	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.21808 β=647.5
Gumbel	0.2400	0.2002	0.1805	0.41145	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=235.76 μ=5.1206
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.16339	4	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.1717 μ=2.0678
Normal	0.2400	0.2002	0.1805	0.34327	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=302.37 μ=141.2
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.15355	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36467 β=37.261

Figure B.35 Fitted gamma distribution and histogram for Oklahoma

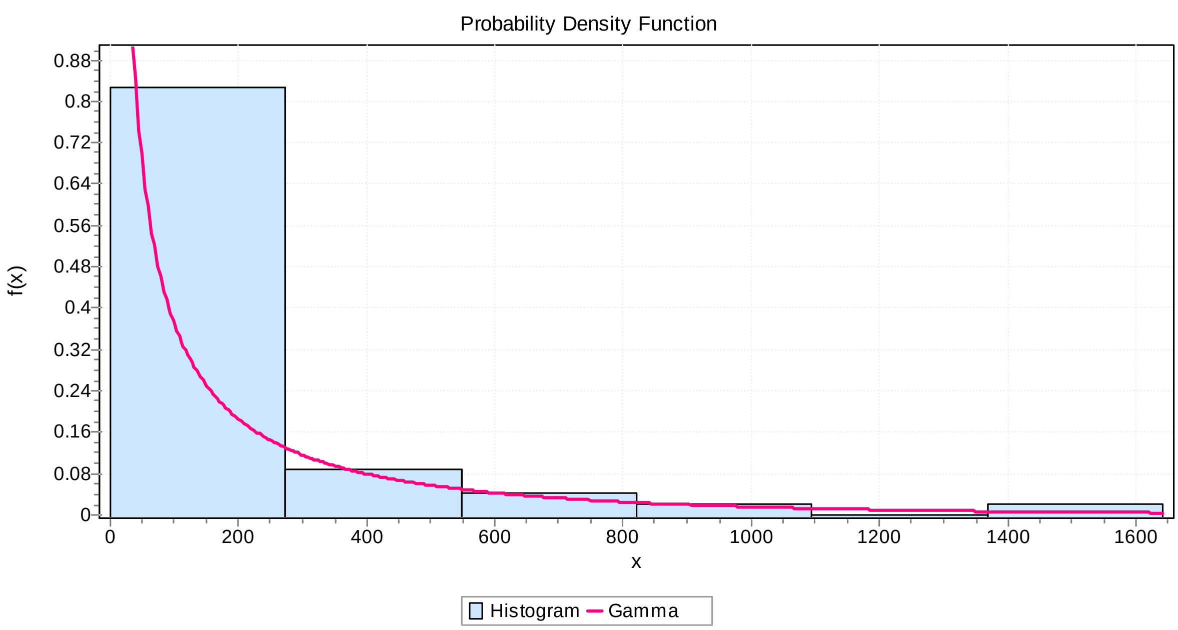


Table B.36 Goodness-of-fit and Distribution Parameters (Oregon)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.57411	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.03998 $\alpha_{2=}$ 0.51667
Frechet	0.2400	0.2002	0.1805	0.19927	3	Fail to Reject H_0 at α=0.01,0.05	α=0.36205 β=1.455
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.33598	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.09995 β=2647.5
Gumbel	0.2400	0.2002	0.1805	0.48226	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=652.6 μ=-112.08
Lognormal	0.2400	0.2002	0.1805	0.12193	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.1612 μ=2.0246
Normal	0.2400	0.2002	0.1805	0.4129	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=836.99 μ=264.61
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.09211	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.33837 β=36.15

Figure B.36 Fitted Weibull distribution and histogram for Oregon

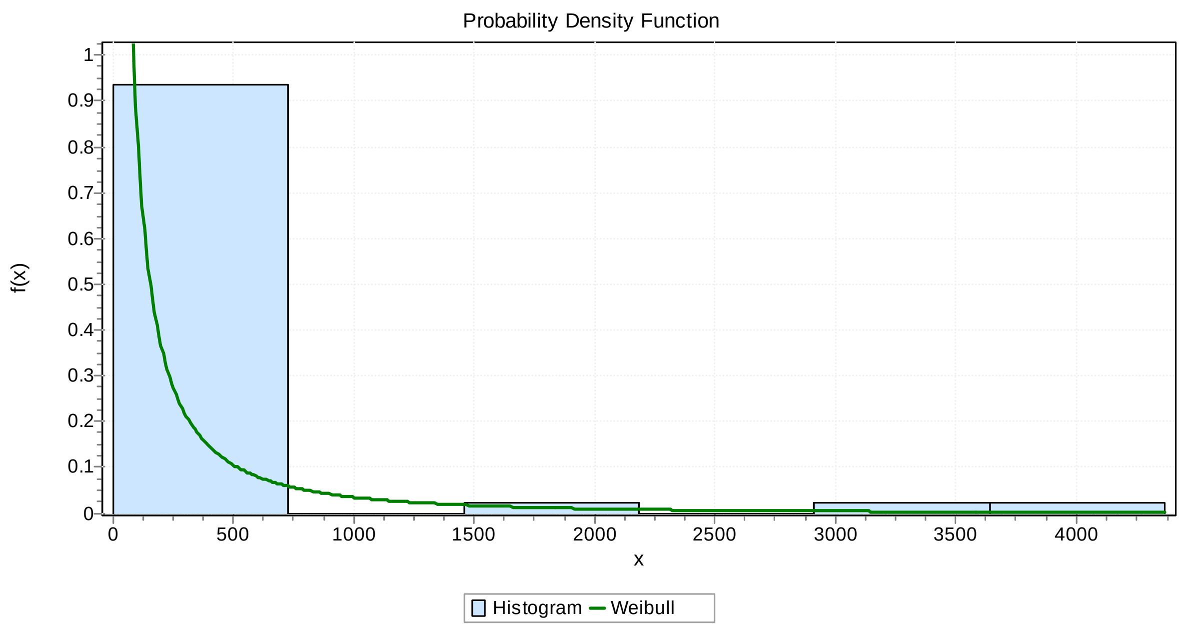


Table B.37 Goodness-of-fit and Distribution Parameters (Pennsylvania)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.80176	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.01217 $\alpha_{2=}$ 0.33168
Frechet	0.2400	0.2002	0.1805	0.17123	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.35102 β=1.9519
Gamma	0.2400	0.2002	0.1805	0.60094	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.04102 β=22789.0
Gumbel	0.2400	0.2002	0.1805	0.51633	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=3598.8 μ=-1142.4
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.11213	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.337 μ=2.4029
Normal	0.2400	0.2002	0.1805	0.44574	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=4615.6 μ=934.83
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.11045	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.30622 β=59.187

Figure B.37 Fitted Weibull distribution and histogram for Pennsylvania

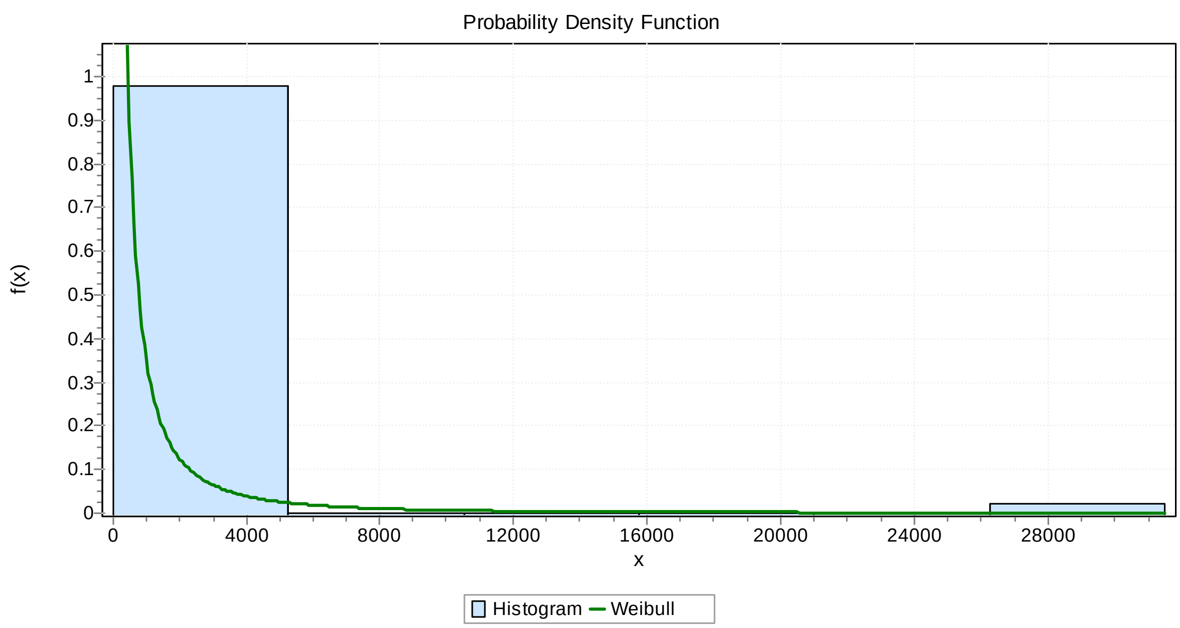


Table B.38 Goodness-of-fit and Distribution Parameters (Rhode Island)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.89275	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.00517 $\alpha_{2=}$ 0.16262
Frechet	0.2400	0.2002	0.1805	0.44119	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.58813 β=0.05186
Gamma	0.2400	0.2002	0.1805	0.72122	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.03093 β=766.62
Gumbel	0.2400	0.2002	0.1805	0.57122	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=105.12 μ=-36.967
Lognormal	0.2400	0.2002	0.1805	0.46744	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=1.8918 μ=-1.8347
Normal	0.2400	0.2002	0.1805	0.50198	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=134.83 μ=23.714
Weibull	0.2400	0.2002	0.1805	0.42495	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.3223 β=0.51817

Figure B.38 Fitted Weibull distribution and histogram for Rhode Island

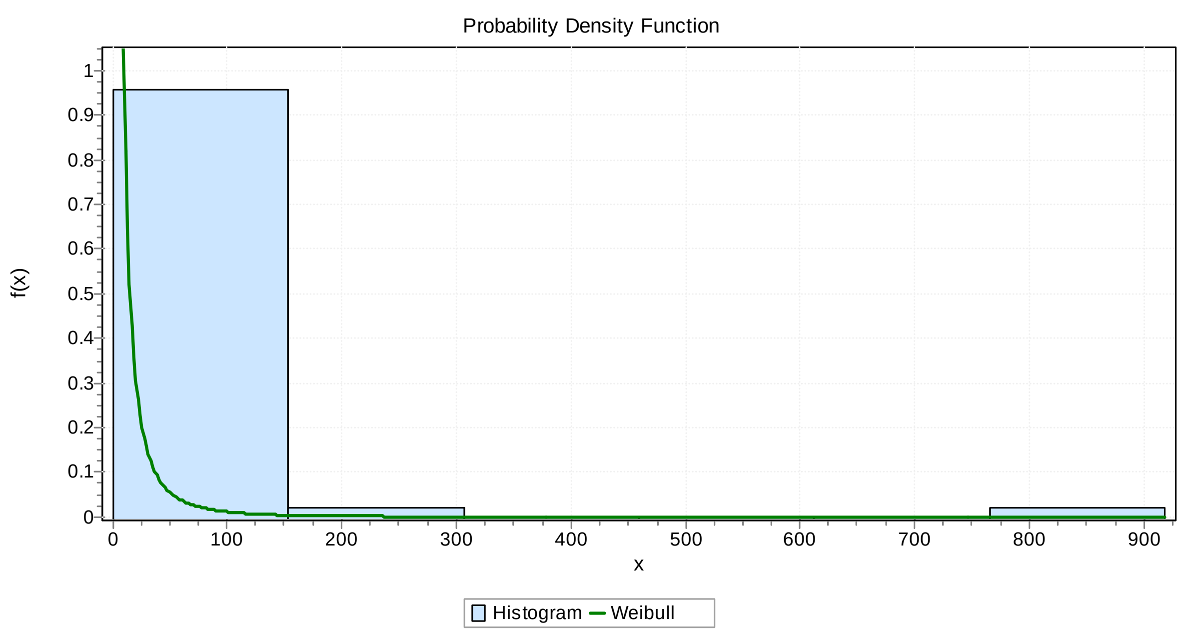


Table B.39 Goodness-of-fit and Distribution Parameters (South Carolina)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.68266	7	Reject H_0 at α=0.01,0.05 & 0.1	α_1= 0.03277 α_2= 0.68169
Frechet	0.2400	0.2002	0.1805	0.1604	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.52063 β=0.65812
Gamma	0.2400	0.2002	0.1805	0.57912	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.06889 β=340.18
Gumbel	0.2400 ***	0.2002 **	0.1805 *	0.44536	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=69.616 μ=-16.749
Lognormal	0.2400	0.2002	0.1805	0.14486	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.2567 μ=0.75685
Normal	0.2400 ***	0.2002 **	0.1805 *	0.39679	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=89.286 μ=23.435
Weibull	0.2400	0.2002	0.1805	0.12549	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.44466 β=6.6648

Figure B.39 Fitted Weibull distribution and histogram for South Carolina

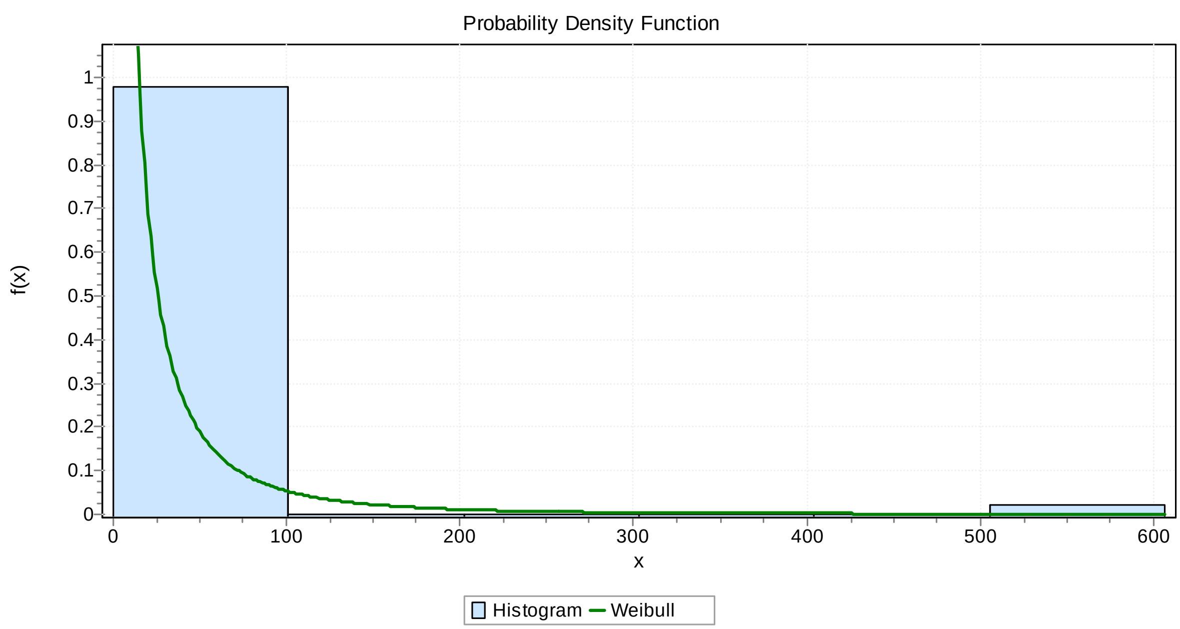


Table B.40 Goodness-of-fit and Distribution Parameters (South Dakota)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.24681	1	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.04464 $\alpha_{2=}$ 0.82388
Frechet	0.2400	0.2002	0.1805	0.32575	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.30783 β=0.25174
Gamma	0.2400	0.2002	0.1805	0.32575	2	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.09901 β=1088.5
Gumbel	0.2400	0.2002	0.1805	0.4526	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=267.06 μ=-46.376
Lognormal	0.2400	0.2002	0.1805	0.35003	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.7839 μ=1.1993
Normal	0.2400	0.2002	0.1805	0.38429	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=342.51 μ=107.77
Weibull	0.2400	0.2002	0.1805	0.32575	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.33971 β=30.213

Figure B.40 Fitted Beta distribution and histogram for South Dakota

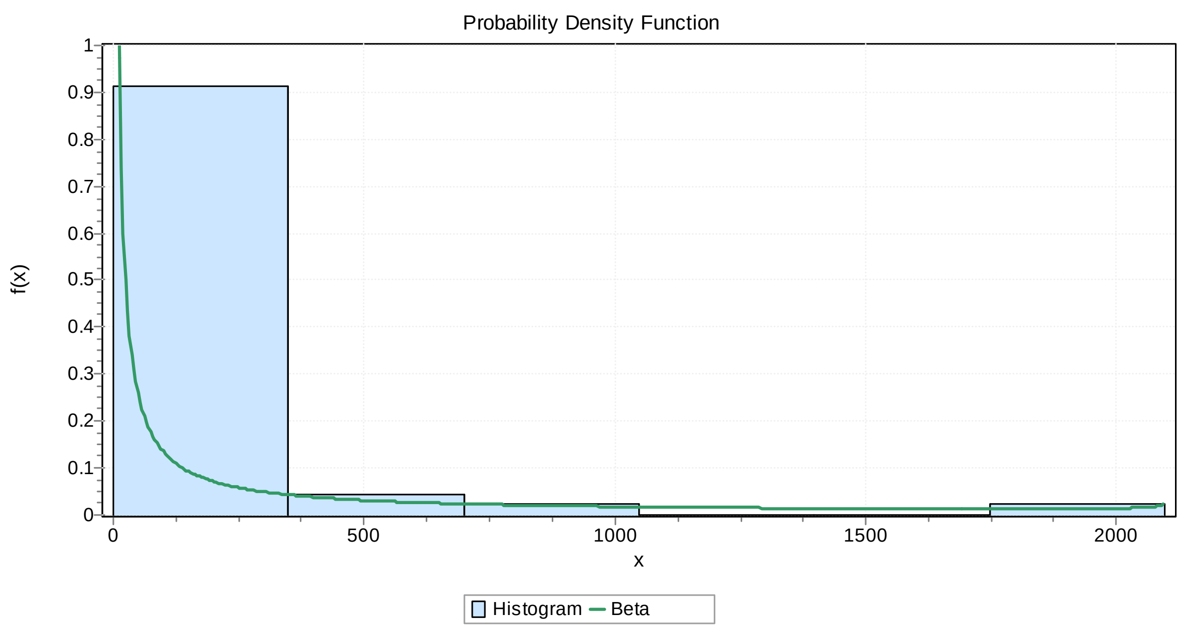


Table B.41 Goodness-of-fit and Distribution Parameters (Tennessee)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.28664	5	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.10145 $\alpha_{2=}$ 0.72314
Frechet	0.2400	0.2002	0.1805	0.17951	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.42831 β=2.5053
Gamma	*** 0.2400	** 0.2002	0.1805	0.18776	4	Fail to Reject H_0 at α=0.01&0.05 and Reject H_0 at α=0.1	α= 0.21182 β=408.49
Gumbel	0.2400	0.2002	0.1805	0.38594	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=146.58 μ=1.9141
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.11687	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5857 μ=2.292
Normal	0.2400	0.2002	0.1805	0.32286	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=188.0 μ=86.524
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.06976	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.45004 β=34.267

Figure B.41 Fitted Weibull distribution and histogram for Tennessee

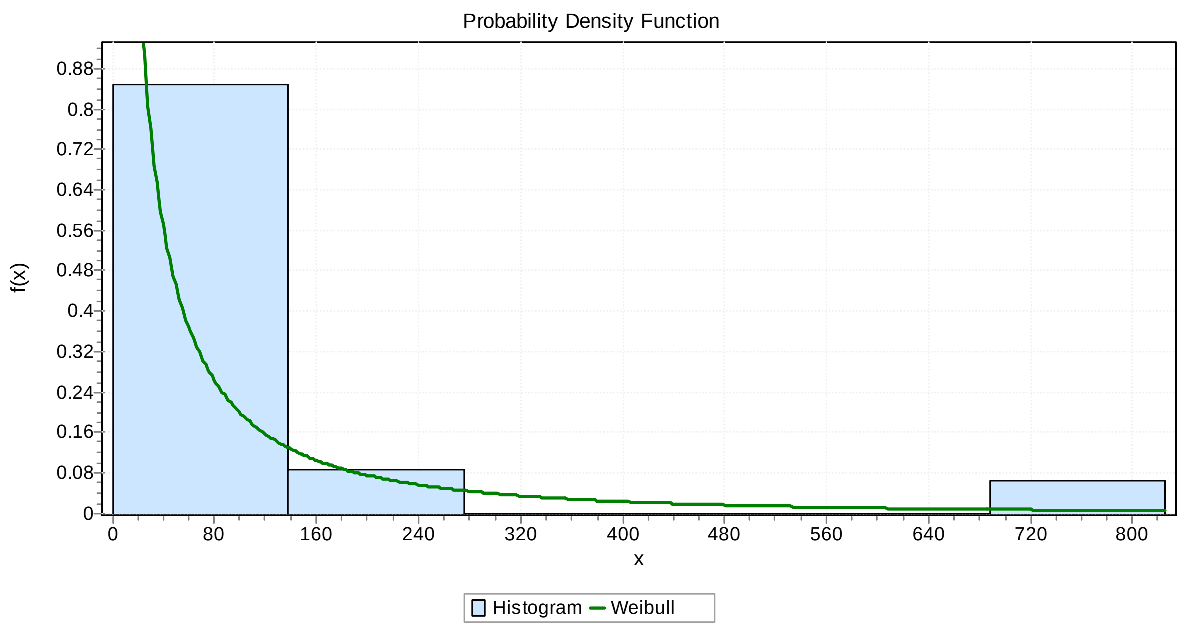


Table B.42 Goodness-of-fit and Distribution Parameters (Texas)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.34577	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.12425 $\alpha_{2=}$ 1.5681
Frechet	0.2400	0.2002	0.1805	0.28706	4	Reject H_0 at α=0.01,0.05 & 0.1	α=0.33115 β=14.935
Gamma	0.2400	0.2002	0.1805	0.24639	3	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.20094 β=3249.3
Gumbel	0.2400	0.2002	0.1805	0.38043	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=1135.7 μ=-2.6089
Lognormal	*** 0.2400	0.2002	0.1805	0.20185	2	Fail to Reject H_0 at α=0.01&Reject H_0 at α=0.05 & 0.1	σ=2.9776 μ=4.4569
Normal	0.2400	0.2002	0.1805	0.32701	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=1456.6 μ=652.92
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.11666	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.47334 β=314.63

Figure B.42 Fitted Weibull distribution and histogram for Texas

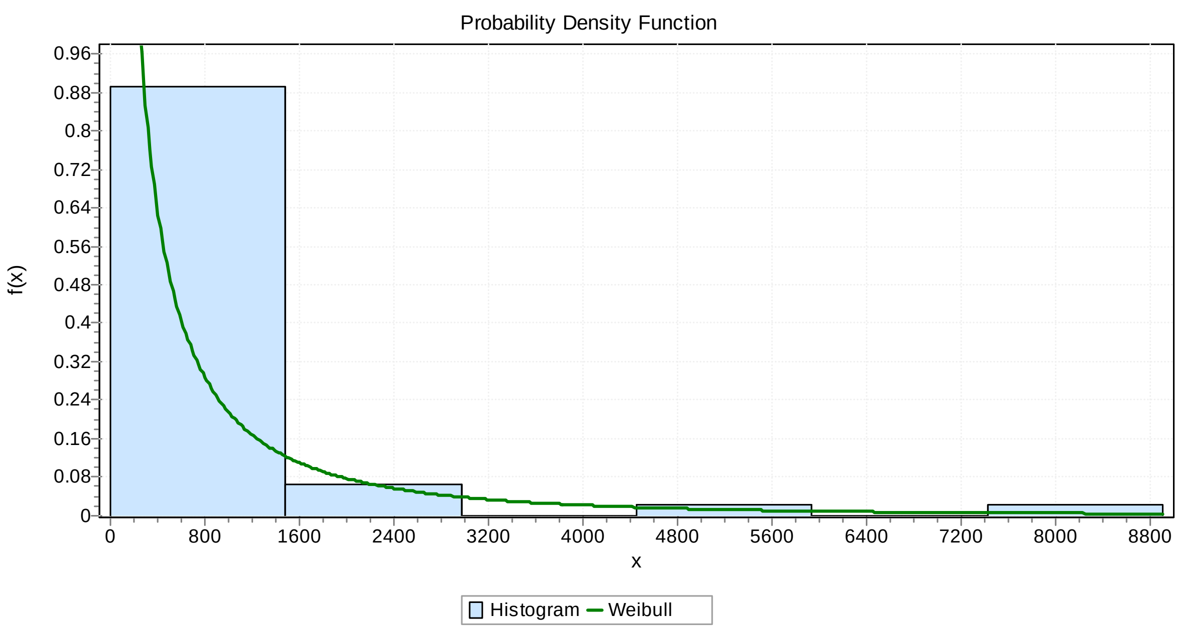


Table B.43 Goodness-of-fit and Distribution Parameters (Utah)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.77512	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.01758$\alpha_{2=}$ 0.30004
Frechet	0.2400	0.2002	0.1805	0.15377	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.45901 β=0.50512
Gamma	0.2400	0.2002	0.1805	0.548	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.06427 β=892.36
Gumbel	0.2400	0.2002	0.1805	0.53682	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=176.39 μ=-44.462
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.15093	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.5913 μ=0.65106
Normal	0.2400	0.2002	0.1805	0.46623	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=226.23 μ=57.353
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.15512	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.36567 β=7.3315

Figure B.43 Fitted Lognormal distribution and histogram for Utah

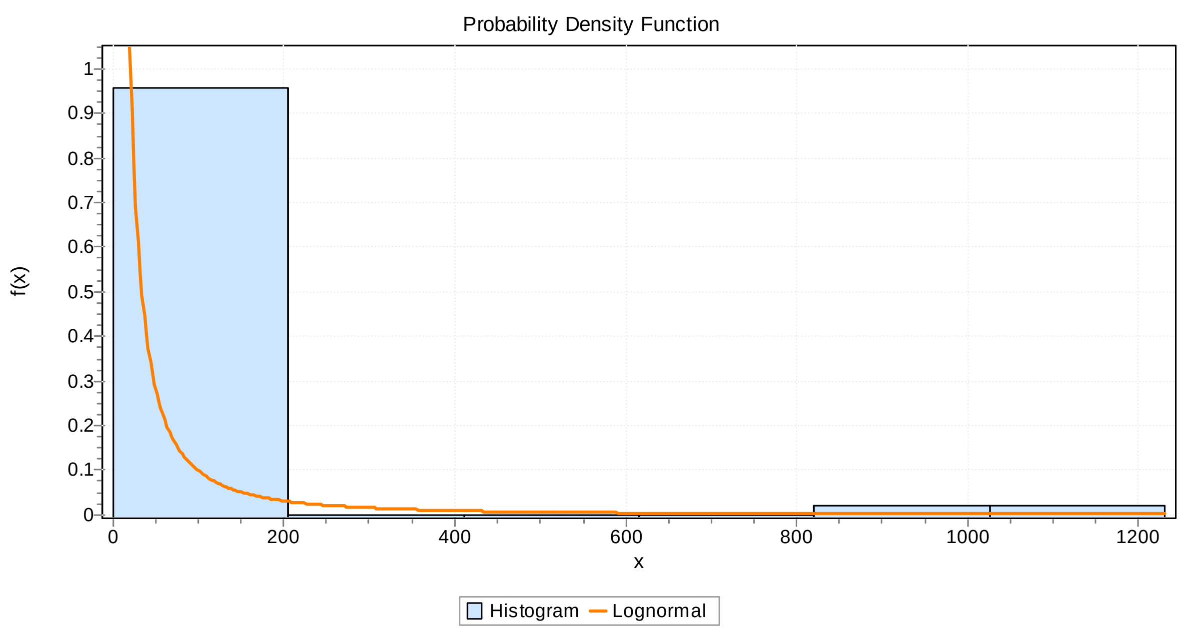


Table B.44 Goodness-of-fit and Distribution Parameters (Vermont)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.82019	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_{1=}$ 0.01194 $\alpha_{2=}$ 0.24812
Frechet	0.2400	0.2002	0.1805	0.26899	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.44507 β=0.11989
Gamma	0.2400	0.2002	0.1805	0.59908	6	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.05046 β=887.64
Gumbel	0.2400	0.2002	0.1805	0.56859	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=155.47 μ=-44.947
Lognormal	0.2400	0.2002	0.1805	0.33161	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.6562 μ=0.71951
Normal	0.2400	0.2002	0.1805	0.49821	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=199.4 μ=44.792
Weibull	0.2400	0.2002	0.1805	0.28753	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.31447 β=2.0776

Figure B.44 Fitted Weibull distribution for Vermont

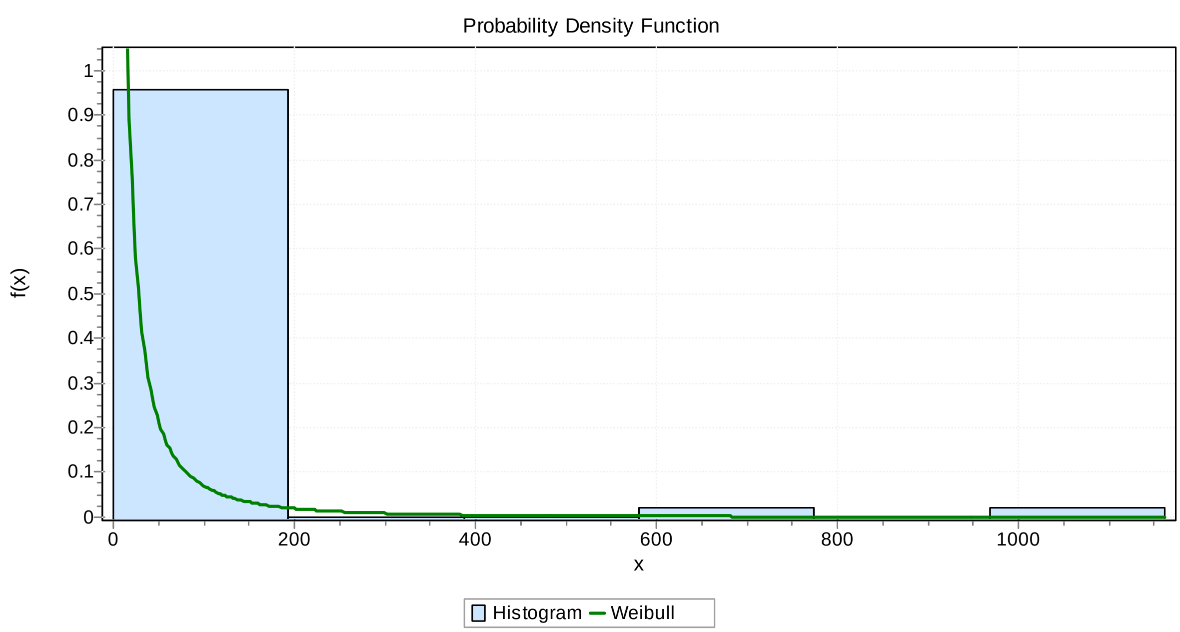


Table B.45 Goodness-of-fit and Distribution Parameters (Virginia)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.43014	6	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_1 =$ 0.06149 $\alpha_2 =$ 0.43206
Frechet	0.2400	0.2002	0.1805	0.17259	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.33639 β=0.82306
Gamma	*** 0.2400	** 0.2002	0.1805	0.18708	4	Fail to Reject H_0 at α=0.01&0.05 and Reject H_0 at α=0.1	α= 0.17599 β=1383.4
Gumbel	0.2400	0.2002	0.1805	0.46325	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=452.52 μ=-17.722
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.17572	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=3.4068 μ=1.5682
Normal	0.2400	0.2002	0.1805	0.3941	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=580.37 μ=243.48
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.13564	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.30134 β=27.55

Figure B.45 Fitted Weibull distribution and histogram for Virginia

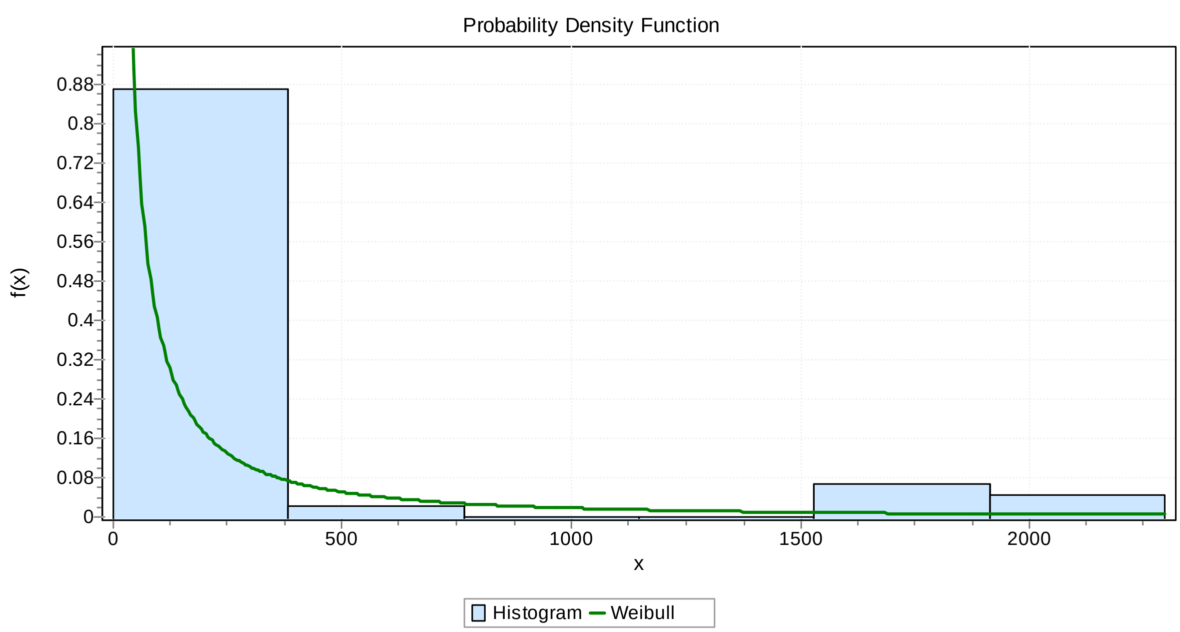


Table B.46 Goodness-of-fit and Distribution Parameters (Washington)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	** 0.2002	0.1805	0.19782	5	Fail to Reject H_0 at α=0.01&0.05 and Reject H_0 at α= 0.1	$\alpha_{1=}$ 0.1736 $\alpha_{2=}$ 0.86217
Frechet	0.2400	0.2002	0.1805	0.18501	4	Fail to Reject H_0 at α=0.01,0.05	α=0.37351 β=1.2695
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.16112	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.35478 β=170.46
Gumbel	0.2400	0.2002	0.1805	0.38572	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=79.162 μ=14.781
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.14139	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.8871 μ=1.792
Normal	0.2400	0.2002	0.1805	0.31512	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=101.53 μ=60.474
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.12295	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.41908 β=24.036

Figure B.46 Fitted Weibull distribution and histogram for Washington

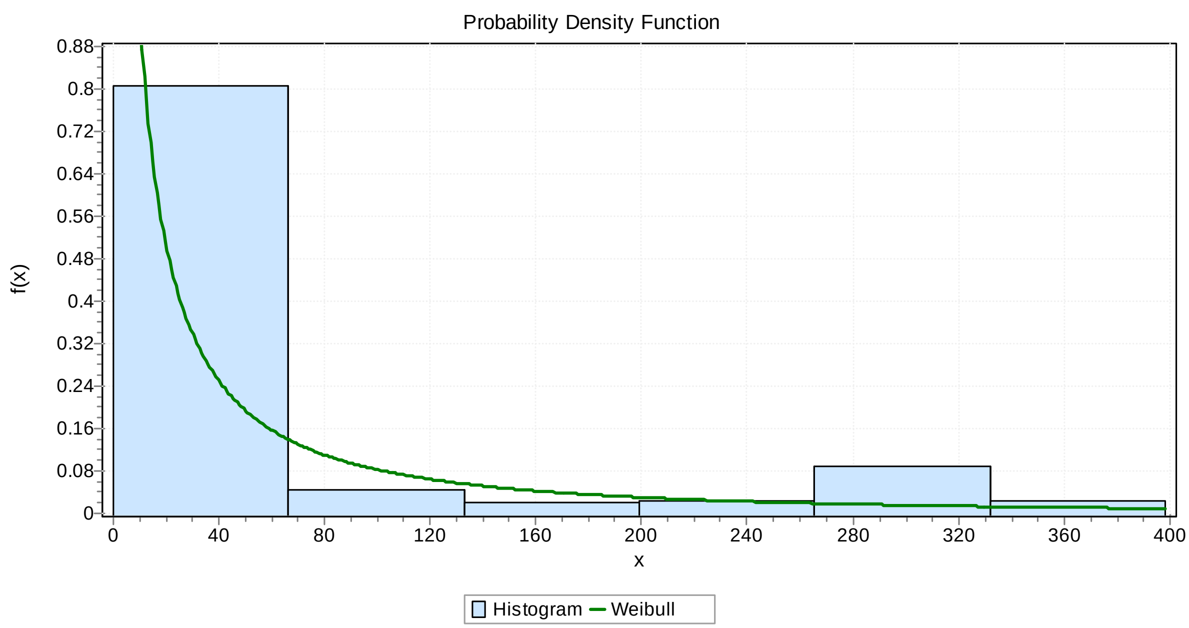


Table B.47 Goodness-of-fit and Distribution Parameters (West Virginia)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	*** 0.2400	** 0.2002	0.1805	0.1929	5	Fail to Reject H_0 at α=0.01 & 0.05 and Reject H_0 at α=0.1	α_1=0.17334 α_2= 1.5005
Frechet	0.2400	0.2002	0.1805	0.19007	4	Fail to Reject H_0 at α=0.01,0.05	α=0.37248 β=2.4848
Gamma	*** 0.2400	** 0.2002	* 0.1805	0.10984	2	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α= 0.25464 β=423.49
Gumbel	0.2400	0.2002	0.1805	0.36844	7	Reject H_0 at α=0.01,0.05 & 0.1	σ=166.62 μ=11.66
Lognormal	*** 0.2400	** 0.2002	* 0.1805	0.11026	3	Fail to Reject H_0 at α=0.01,0.05 & 0.1	σ=2.8608 μ=2.4794
Normal	0.2400	0.2002	0.1805	0.30692	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=213.7 μ=107.84
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.06315	1	Fail to Reject H_0 at α=0.01,0.05 & 0.1	α=0.44072 β=44.712

Figure B.47 Fitted Weibull distribution and histogram for West Virginia

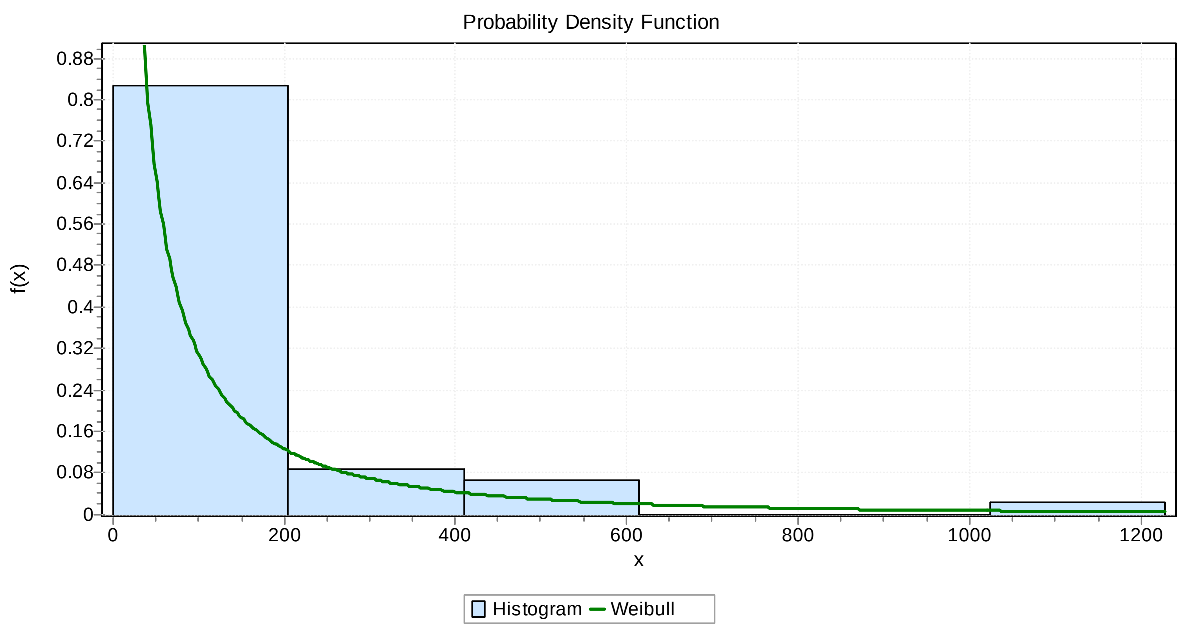


Table B.48 Goodness-of-fit and Distribution Parameters (Wisconsin)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.42755	7	Reject H_0 at α=0.01,0.05 & 0.1	$\alpha_1=$ 0.04447 $\alpha_2=$ 0.50152
Frechet	0.2400	0.2002	0.1805	0.19645	2	Fail to Reject H_0 at α=0.01,0.05	α=0.36293 β=0.58021
Gamma	0.2400	0.2002	0.1805	0.32797	4	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.11439 β=52.889
Gumbel	0.2400	0.2002	0.1805	0.39317	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=137.61 μ=-15.546
Lognormal	*** 0.2400	0.2002	0.1805	0.21393	3	Fail to Reject H_0 at α=0.01& Reject H_0 at α=0.05& 0.1	σ=2.2137 μ=0.90321
Normal	0.2400	0.2002	0.1805	0.35881	5	Reject H_0 at α=0.01,0.05 & 0.1	σ=17.888 μ=6.05
Weibull	*** 0.2400	** 0.2002	* 0.1805	0.18814	1	Fail to Reject H_0 at α=0.01,0.05	α=0.40496 β=1.3386

Figure B.48 Fitted Weibull distribution and histogram for Wisconsin

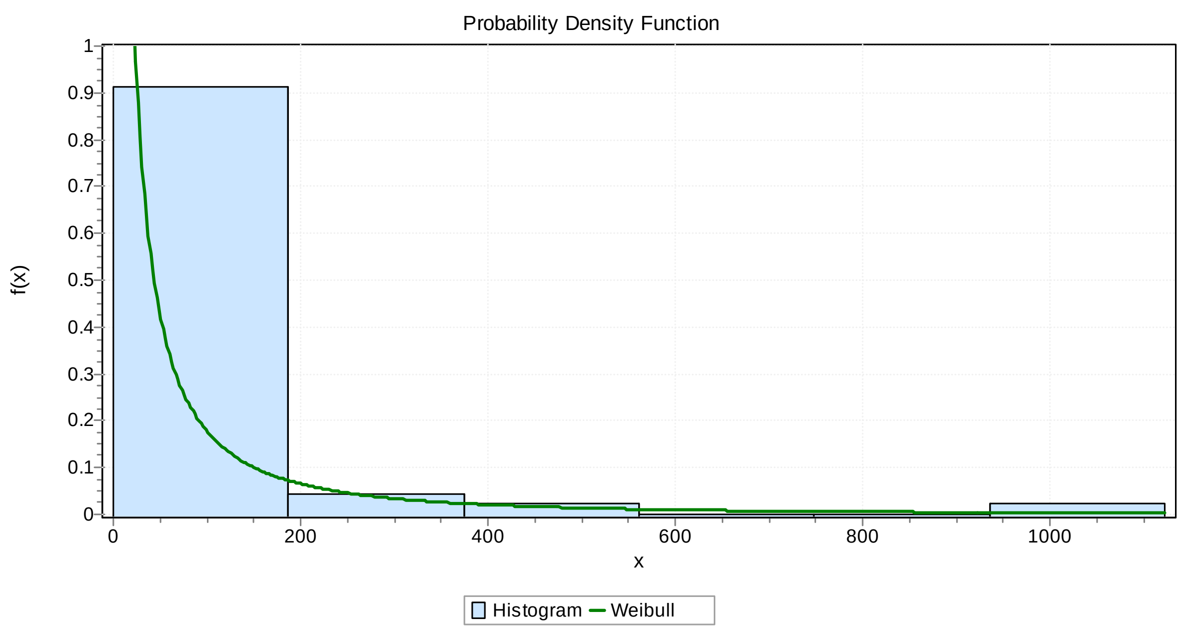


Table B.49 Goodness-of-fit and Distribution Parameters (Wyoming)

Distribution	α=0.01	α=0.05	α=0.1	K - S Statistic	Rank	Reject/Accept	Parameters
Beta	0.2400	0.2002	0.1805	0.60935	7	Reject H_0 at α=0.01,0.05 & 0.1	α_1= 0.03579 α_2= 0.53199
Frechet	0.2400	0.2002	0.1805	0.24437	1	Reject H_0 at α=0.01,0.05 & 0.1	α=0.52568 β=0.12718
Gamma	0.2400	0.2002	0.1805	0.42604	5	Reject H_0 at α=0.01,0.05 & 0.1	α= 0.08763 β=769.57
Gumbel	0.2400	0.2002	0.1805	0.43299	6	Reject H_0 at α=0.01,0.05 & 0.1	σ=13.947 μ=-2.0005
Lognormal	0.2400	0.2002	0.1805	0.30223	3	Reject H_0 at α=0.01,0.05 & 0.1	σ=2.7399 μ=0.52298
Normal	0.2400	0.2002	0.1805	0.36799	4	Reject H_0 at α=0.01,0.05 & 0.1	σ=227.81 μ=67.434
Weibull	0.2400	0.2002	0.1805	0.27075	2	Reject H_0 at α=0.01,0.05 & 0.1	α=0.33919 β=7.1136

Figure B.49 Fitted Weibull distribution and histogram for Wyoming

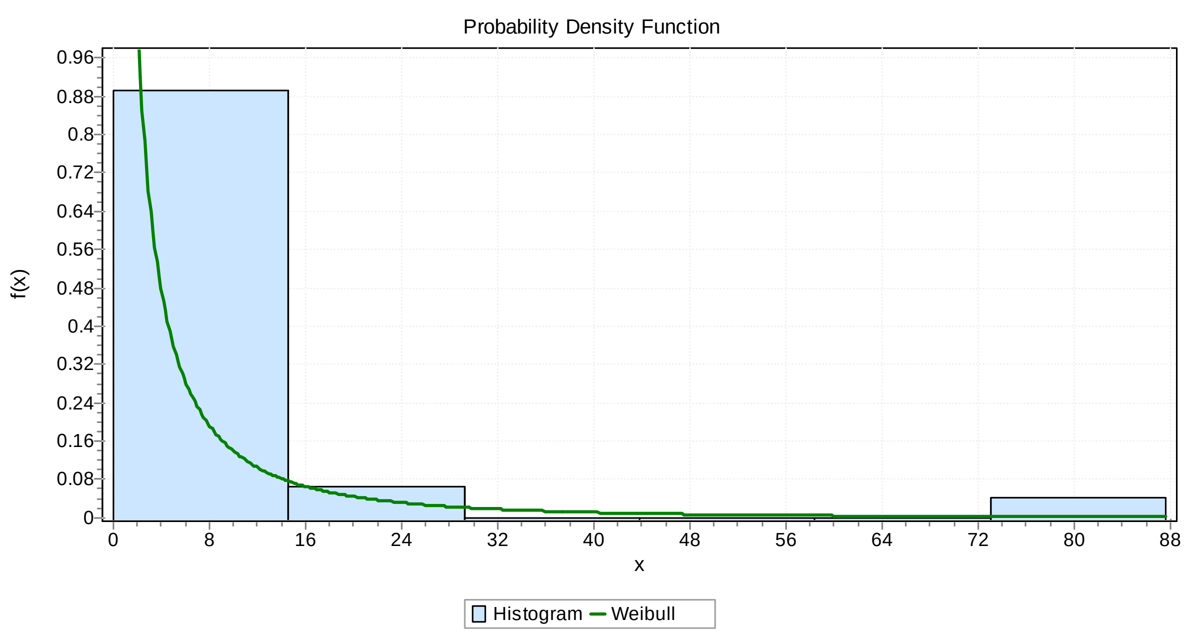


Table B.50 Goodness-of-fit and Distribution Parameters (Alaska)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	N/A	N/A	N/A	α_1=0.00312 α_2=0.12486
Frechet	3.9074	2.5018	1.9286	6.5562	1	Reject H_0 at .01, .05, .1	α=0.49141 β=0.10785
Gamma	3.9074	2.5018	1.9286	33.016	6	Reject H_0 at .01, .05, .1	α=0.02789 β=1720.6
Gumbel	3.9074	2.5018	1.9286	15.507	4	Reject H_0 at .01, .05, .1	μ=224.05 σ=-81.334
Lognormal	3.9074	2.5018	1.9286	7.6845	3	Reject H_0 at .01, .05, .1	σ=2.3958 μ=-0.92416
Normal	3.9074	2.5018	1.9286	16.292	5	Reject H_0 at .01, .05, .1	σ=287.35 μ=47.99
Weibull	3.9074	2.5018	1.9286	7.1517	2	Reject H_0 at .01, .05, .1	α=0.31815 β=1.543

Table B.51 Goodness-of-fit and distribution Parameters (Arizona)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.108	4	Reject H_0 at .01, .05, .1	α_1=0.10565 α_2=0.85215
Frechet	3.9074	2.5018	1.9286	1.7603	3	Fail to reject H_0 at .01, .05, .1	α=0.36528 β=0.59276
Gamma	3.9074	2.5018	1.9286	1.6816	2	Fail to reject H_0 at .01, .05, .1	α=0.20053 β=329.58
Gumbel	3.9074	2.5018	1.9286	7.8589	6	Reject H_0 at .01, .05, .1	μ=115.07 σ=-0.33213
Lognormal	3.9074	2.5018	1.9286	1.5174	1	Fail to reject H_0 at .01, .05, .1	σ=3.0796 μ=1.0949
Normal	3.9074	2.5018	1.9286	9.3794	7	Reject H_0 at .01, .05, .1	σ=147.59 μ=66.09
Weibull	3.9074	2.5018	1.9286	4.9351	5	Reject H_0 at .01, .05, .1	α=0.3514 β=14.039

Table B.52 Goodness-of-fit and Distribution Parameters (Arkansas)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	7.9559	5	Reject H_0 at .01, .05, .1	α_1=0.07626 α_2=0.74122
Frechet	3.9074	2.5018	1.9286	1.6853	3	Fail to reject H_0 at .01, .05, .1	α=0.44867 β=2.9167
Gamma	3.9074	2.5018	1.9286	4.0634	4	Reject H_0 at .01, .05, .1	α=0.15501 β=748.1
Gumbel	3.9074	2.5018	1.9286	8.8481	6	Reject H_0 at .01, .05, .1	μ=229.65 σ=-16.595
Lognormal	3.9074	2.5018	1.9286	0.38291	1	Fail to reject H_0 at .01, .05, .1	σ=2.5344 μ=2.3967
Normal	3.9074	2.5018	1.9286	10.459	7	Reject H_0 at .01, .05, .1	σ=294.53 μ=115.96
Weibull	3.9074	2.5018	1.9286	0.38938	2	Fail to reject H_0 at .01, .05, .1	α=0.43347 β=37.938

Table B.53 Goodness-of-fit and Distribution Parameters (California)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	3.7179	5	Reject H_0 at .05, .1	α_1=0.12378 α_2=1.2044
Frechet	3.9074	2.5018	1.9286	2.8082	4	Reject H_0 at .05, .1	α=0.30267 β=6.7467
Gamma	3.9074	2.5018	1.9286	0.61305	1	Fail to reject H_0 at .01, .05, .1	α=0.26245 β=2954.6
Gumbel	3.9074	2.5018	1.9286	6.0016	6	Reject H_0 at .01, .05, .1	σ=1180.2 μ=94.21
Lognormal	3.9074	2.5018	1.9286	1.4964	3	Fail to reject H_0 at .01, .05, .1	σ=3.494 μ=3.8289
Normal	3.9074	2.5018	1.9286	7.6197	7	Reject H_0 at .01, .05, .1	σ=1513.6 μ=775.42
Weibull	3.9074	2.5018	1.9286	0.73274	2	Fail to reject H_0 at .01, .05, .1	α=0.36534 β=235.65

Table B.54 Goodness-of-fit and Distribution Parameters (Colorado)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	43.11	7	Reject H_0 at .01, .05, .1	α_1=0.01376 α_2=0.366
Frechet	3.9074	2.5018	1.9286	1.3995	1	Fail to reject H_0 at .01, .05, .1	α=0.36624 β=0.36258
Gamma	3.9074	2.5018	1.9286	17.765	6	Reject H_0 at .01, .05, .1	α=0.04317 β=7179.2
Gumbel	3.9074	2.5018	1.9286	13.48	4	Reject H_0 at .01, .05, .1	σ=1163.1 μ=-361.4
Lognormal	3.9074	2.5018	1.9286	2.1522	2	Reject H_0 at .1	σ=3.2801 μ=0.66769
Normal	3.9074	2.5018	1.9286	14.29	5	Reject H_0 at .01, .05, .1	σ=1491.7 μ=309.95
Weibull	3.9074	2.5018	1.9286	2.7087	3	Reject H_0 at .05, .1	α=0.27682 β=11.274

Table B.55 Goodness-of-fit and Distribution Parameters (Connecticut)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	103.34	7	Reject H_0 at .01, .05, .1	α_1=0.0014 α_2=0.05033
Frechet	3.9074	2.5018	1.9286	3.6226	1	Reject H_0 at .05, .1	α=0.42308 β=0.12988
Gamma	3.9074	2.5018	1.9286	31.154	6	Reject H_0 at .01, .05, .1	α=0.0243 β=11237.0
Gumbel	3.9074	2.5018	1.9286	16.112	4	Reject H_0 at .01, .05, .1	σ=1365.7 μ=-515.28
Lognormal	3.9074	2.5018	1.9286	4.4942	2	Reject H_0 at .01, .05, .1	σ=2.9226 μ=-0.52569
Normal	3.9074	2.5018	1.9286	16.869	5	Reject H_0 at .01, .05, .1	σ=1751.6 μ=273.05
Weibull	3.9074	2.5018	1.9286	4.7135	3	Reject H_0 at .01, .05, .1	α=0.27401 β=2.9717

Table B.56 Goodness-of-fit and Distribution Parameters (Delaware)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	36.281	7	Reject H_0 at .01, .05, .1	α_1=0.0373 α_2=0.69685
Frechet	3.9074	2.5018	1.9286	9.9934	1	Reject H_0 at .01, .05, .1	α=1.0171 β=0.07304
Gamma	3.9074	2.5018	1.9286	26.76	6	Reject H_0 at .01, .05, .1	α=0.08564 β=5.967
Gumbel	3.9074	2.5018	1.9286	13.111	4	Reject H_0 at .01, .05, .1	σ=1.3615 μ=-0.27486
Lognormal	3.9074	2.5018	1.9286	11.125	3	Reject H_0 at .01, .05, .1	σ=1.0792 μ=-1.9796
Normal	3.9074	2.5018	1.9286	13.955	5	Reject H_0 at .01, .05, .1	σ=1.7462 μ=0.51104
Weibull	3.9074	2.5018	1.9286	11.023	2	Reject H_0 at .01, .05, .1	α=0.61771 β=0.26398

Table B.57 Goodness-of-fit and Distribution Parameters (Florida)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	5.6493	5	Reject H_0 at .01, .05, .1	α_1=0.12455 α_2=1.01
Frechet	3.9074	2.5018	1.9286	2.1514	4	Reject H_0 at .1	α=0.40319 β=1.7103
Gamma	3.9074	2.5018	1.9286	0.73108	2	Fail to reject H_0 at .01, .05, .1	α=0.29183 β=231.47
Gumbel	3.9074	2.5018	1.9286	5.8126	6	Reject H_0 at .01, .05, .1	σ=97.497 μ=11.274
Lognormal	3.9074	2.5018	1.9286	1.0494	3	Fail to reject H_0 at .01, .05, .1	σ=2.7166 μ=1.9869
Normal	3.9074	2.5018	1.9286	7.5772	7	Reject H_0 at .01, .05, .1	σ=125.04 μ=67.551
Weibull	3.9074	2.5018	1.9286	0.63002	1	Fail to reject H_0 at .01, .05, .1	α=0.43482 β=27.04

Table B.58 Goodness-of-fit and Distribution Parameters (Georgia)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	5.0103	5	Reject H_0 at .01, .05, .1	α_1=0.12799 α_2=1.251
Frechet	3.9074	2.5018	1.9286	2.0239	3	Reject H_0 at.1	α=0.46065 β=0.94093
Gamma	3.9074	2.5018	1.9286	2.3038	4	Reject H_0 at.1	α=0.20146 β=139.03
Gumbel	3.9074	2.5018	1.9286	7.2109	6	Reject H_0 at .01, .05, .1	σ=48.655 μ=-0.07538
Lognormal	3.9074	2.5018	1.9286	0.88152	2	Fail to reject H_0 at .01, .05, .1	σ=2.4362 μ=1.2276
Normal	3.9074	2.5018	1.9286	8.5153	7	Reject H_0 at .01, .05, .1	σ=62.403 μ=28.009
Weibull	3.9074	2.5018	1.9286	0.61215	1	Fail to reject H_0 at .01, .05, .1	α=0.45931 β=11.242

Table B.59 Goodness-of-fit and Distribution Parameters (Hawaii)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	N/A	N/A	N/A	α_1=0.02509 α_2=0.65523
Frechet	3.9074	2.5018	1.9286	4.2779	2	Reject H_0 at .01, .05, .1	α=0.44853 β=0.18867
Gamma	3.9074	2.5018	1.9286	17.228	6	Reject H_0 at .01, .05, .1	α=0.06276 β=279.3
Gumbel	3.9074	2.5018	1.9286	11.725	4	Reject H_0 at .01, .05, .1	σ=54.553 μ=-13.961
Lognormal	3.9074	2.5018	1.9286	4.9702	3	Reject H_0 at .01, .05, .1	σ=2.512 μ=-0.30331
Normal	3.9074	2.5018	1.9286	12.616	5	Reject H_0 at .01, .05, .1	σ=69.967 μ=17.528
Weibull	3.9074	2.5018	1.9286	4.2453	1	Reject H_0 at .01, .05, .1	α=0.37054 β=2.8071

Table B.60 Goodness-of-fit and Distribution Parameters (Idaho)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	40.776	7	Reject H_0 at .01, .05, .1	α_1=0.01625 α_2=0.41534
Frechet	3.9074	2.5018	1.9286	1.4473	1	Fail to reject H_0 at .01, .05, .1	α=0.38104 β=0.42294
Gamma	3.9074	2.5018	1.9286	18.538	6	Reject H_0 at .01, .05, .1	α=0.04662 β=3091.2
Gumbel	3.9074	2.5018	1.9286	13.087	4	Reject H_0 at .01, .05, .1	σ=520.39 μ=-156.27
Lognormal	3.9074	2.5018	1.9286	1.7736	2	Fail to reject H_0 at .01, .05, .1	σ=3.1032 μ=0.74413
Normal	3.9074	2.5018	1.9286	13.945	5	Reject H_0 at .01, .05, .1	σ=667.42 μ=144.1
Weibull	3.9074	2.5018	1.9286	1.9573	3	Fail to reject H_0 at .01, .05	α=0.30944 β=10.668

Table B.61 Goodness-of-fit and Distribution Parameters (Illinois)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	3.4176	5	Fail to reject H_0 at .01	α_1=0.11616 α_2=0.69897
Frechet	3.9074	2.5018	1.9286	1.8475	4	Fail to reject H_0 at .01, .05, .1	α=0.386 β=6.8325
Gamma	3.9074	2.5018	1.9286	1.1067	3	Fail to reject H_0 at .01, .05, .1	α=0.24882 β=1604.7
Gumbel	3.9074	2.5018	1.9286	6.9078	6	Reject H_0 at .01, .05, .1	σ=624.12 μ=39.033
Lognormal	3.9074	2.5018	1.9286	0.56833	1	Fail to reject H_0 at .01, .05, .1	σ=2.8733 μ=3.4395
Normal	3.9074	2.5018	1.9286	8.4408	7	Reject H_0 at .01, .05, .1	σ=800.46 μ=399.28
Weibull	3.9074	2.5018	1.9286	0.57577	2	Fail to reject H_0 at .01, .05, .1	α=0.40103 β=124.77

Table B.62 Goodness-of-fit and Distribution Parameters (Indiana)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.3473	5	Reject H_0 at .01, .05, .1	α_1=0.16043 α_2=1.3813
Frechet	3.9074	2.5018	1.9286	3.4666	4	Fail to reject H_0 at .01	α=0.41605 β=7.9147
Gamma	3.9074	2.5018	1.9286	1.0673	2	Fail to reject H_0 at .01, .05, .1	α=0.27262 β=705.15
Gumbel	3.9074	2.5018	1.9286	5.8454	6	Reject H_0 at .01, .05, .1	σ=287.06 μ=26.536
Lognormal	3.9074	2.5018	1.9286	1.3071	3	Fail to reject H_0 at .01, .05, .1	σ=2.5273 μ=3.4747
Normal	3.9074	2.5018	1.9286	7.4316	7	Reject H_0 at .01, .05, .1	σ=368.18 μ=192.23
Weibull	3.9074	2.5018	1.9286	0.31924	1	Fail to reject H_0 at .01, .05, .1	α=0.50973 β=101.87

Table B.63 Goodness-of-fit and Distribution Parameters (Iowa)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.4622	5	Reject H_0 at .01, .05, .1	α_1=0.08257 α_2=1.0724
Frechet	3.9074	2.5018	1.9286	1.7716	3	Fail to reject H_0 at .01, .05, .1	α=0.35177 β=3.2904
Gamma	3.9074	2.5018	1.9286	2.7571	4	Fail to reject H_0 at .01	α=0.13745 β=3129.6
Gumbel	3.9074	2.5018	1.9286	8.7923	6	Reject H_0 at .01, .05, .1	σ=904.67 μ=-92.023
Lognormal	3.9074	2.5018	1.9286	0.61907	2	Fail to reject H_0 at .01, .05, .1	σ=3.1979 μ=2.8773
Normal	3.9074	2.5018	1.9286	9.7695	7	Reject H_0 at .01, .05, .1	σ=1160.3 μ=430.16
Weibull	3.9074	2.5018	1.9286	0.50403	1	Fail to reject H_0 at .01, .05, .1	σ=1160.3 μ=430.16

Table B.64 Goodness-of-fit and Distribution Parameters (Kansas)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.0068	5	Reject H_0 at .01, .05, .1	α_1=0.13273 α_2=0.76491
Frechet	3.9074	2.5018	1.9286	2.8239	4	Fail to reject H_0 at .01	α=0.38854 β=3.232
Gamma	3.9074	2.5018	1.9286	0.89679	2	Fail to reject H_0 at .01, .05, .1	α=0.35668 β=327.14
Gumbel	3.9074	2.5018	1.9286	5.1166	6	Reject H_0 at .01, .05, .1	σ=152.34 μ=28.756
Lognormal	3.9074	2.5018	1.9286	1.145	3	Fail to reject H_0 at .01, .05, .1	σ=2.7547 μ=2.6652
Normal	3.9074	2.5018	1.9286	6.7998	7	Reject H_0 at .01, .05, .1	σ=195.38 μ=116.69
Weibull	3.9074	2.5018	1.9286	0.49633	1	Fail to reject H_0 at .01, .05, .1	α=0.4517 β=52.377

Table B.65 Goodness-of-fit and Distribution Parameters (Kentucky)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	1.4523	2	Fail to reject H_0 at .01, .05, .1	α_1=0.24354 α_2=1.9092
Frechet	3.9074	2.5018	1.9286	2.7858	4	Fail to reject H_0 at .01	α=0.30512 β=3.5932
Gamma	3.9074	2.5018	1.9286	4.6989	7	Reject H_0 at .01, .05, .1	α=0.43403 β=728.97
Gumbel	3.9074	2.5018	1.9286	3.3938	5	Fail to reject H_0 at .01	σ=374.45 μ=100.25
Lognormal	3.9074	2.5018	1.9286	1.9356	3	Fail to reject H_0 at .01, .05	σ=3.4075 μ=3.1762
Normal	3.9074	2.5018	1.9286	4.4063	6	Reject H_0 at .01, .05, .1	σ=480.25 μ=316.39
Weibull	3.9074	2.5018	1.9286	1.4277	1	Fail to reject H_0 at .01, .05, .1	α=0.38 β=118.44

Table B.66 Goodness-of-fit and Distribution Parameters (Louisiana)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	3.0551	4	Fail to reject H_0 at .01	α_1=0.07604 α_2=0.79962
Frechet	3.9074	2.5018	1.9286	1.2005	2	Fail to reject H_0 at .01, .05, .1	α=0.31782 β=1.5043
Gamma	3.9074	2.5018	1.9286	3.1494	5	Fail to reject H_0 at .01	α=0.1989 β=2711.1
Gumbel	3.9074	2.5018	1.9286	8.7618	6	Reject H_0 at .01, .05, .1	σ=942.72 μ=-4.9287
Lognormal	3.9074	2.5018	1.9286	1.0697	1	Fail to reject H_0 at .01, .05, .1	σ=3.5803 μ=2.2663
Normal	3.9074	2.5018	1.9286	10.251	7	Reject H_0 at .01, .05, .1	σ=1209.1 μ=539.22
Weibull	3.9074	2.5018	1.9286	1.3295	3	Fail to reject H_0 at .01, .05, .1	α=0.29314 β=59.477

Table B.67 Goodness-of-fit and Distribution Parameters (Maine)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	8.3589	6	Reject H_0 at .01, .05, .1	α_1=0.09158 α_2=0.84141
Frechet	3.9074	2.5018	1.9286	4.2864	2	Reject H_0 at .01, .05, .1	α=0.43424 β=0.21975
Gamma	3.9074	2.5018	1.9286	4.2995	3	Reject H_0 at .01, .05, .1	α=0.17715 β=60.039
Gumbel	3.9074	2.5018	1.9286	7.4771	5	Reject H_0 at .01, .05, .1	σ=19.703 μ=-0.73692
Lognormal	3.9074	2.5018	1.9286	4.9759	4	Reject H_0 at .01, .05, .1	σ=2.4881 μ=-0.14273
Normal	3.9074	2.5018	1.9286	8.9253	7	Reject H_0 at .01, .05, .1	σ=25.27 μ=10.636
Weibull	3.9074	2.5018	1.9286	4.1302	1	Reject H_0 at .01, .05, .1	α=0.40767 β=3.1526

Table B.68 Goodness-of-fit and Distribution Parameters (Maryland)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	48.024	7	Reject H_0 at .01, .05, .1	α_1=0.01235 α_2=0.33709
Frechet	3.9074	2.5018	1.9286	2.5774	1	Fail to reject H_0 at .01	α=0.42037 β=0.20151
Gamma	3.9074	2.5018	1.9286	21.527	6	Reject H_0 at .01, .05, .1	α=0.04118 β=2018.2
Gumbel	3.9074	2.5018	1.9286	13.673	4	Reject H_0 at .01, .05, .1	σ=319.33 μ=-101.21
Lognormal	3.9074	2.5018	1.9286	3.5202	2	Fail to reject H_0 at .01	σ=2.8481 μ=-0.12169
Normal	3.9074	2.5018	1.9286	14.493	5	Reject H_0 at .01, .05, .1	σ=409.56 μ=83.113
Weibull	3.9074	2.5018	1.9286	3.8723	3	Fail to reject H_0 at .01	α=0.30436 β=4.1725

Table B.69 Goodness-of-fit and Distribution Parameters (Massachusetts)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	42.342	7	Reject H_0 at .01, .05, .1	α_1=0.01263 α_2=0.33802
Frechet	3.9074	2.5018	1.9286	6.8823	1	Reject H_0 at .01, .05, .1	α=0.37009 β=0.0975
Gamma	3.9074	2.5018	1.9286	19.159	6	Reject H_0 at .01, .05, .1	α=0.04203 β=3572.8
Gumbel	3.9074	2.5018	1.9286	13.9	4	Reject H_0 at .01, .05, .1	σ=571.12 μ=-179.49
Lognormal	3.9074	2.5018	1.9286	8.3413	3	Reject H_0 at .01, .05, .1	σ=3.0012 μ=-0.64624
Normal	3.9074	2.5018	1.9286	14.657	5	Reject H_0 at .01, .05, .1	σ=732.48 μ=150.17
Weibull	3.9074	2.5018	1.9286	7.7848	2	Reject H_0 at .01, .05, .1	α=0.2651 β=2.8652

Table B.70 Goodness-of-fit and Distribution Parameters (Michigan)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	0.23544	2.5018	1.9286	37.84	7	Reject H_0 at .01, .05, .1	α_1=0.01852 α_2=0.38736
Frechet	3.9074	2.5018	1.9286	2.7553	1	Fail to reject H_0 at .01	α=0.45764 β=0.19928
Gamma	3.9074	2.5018	1.9286	16.846	6	Reject H_0 at .01, .05, .1	α=0.05598 β=705.04
Gumbel	3.9074	2.5018	1.9286	13.501	4	Reject H_0 at .01, .05, .1	σ=130.07 μ=-35.606
Lognormal	3.9074	2.5018	1.9286	3.5628	2	Fail to reject H_0 at .01	σ=2.6011 μ=-0.2576
Normal	3.9074	2.5018	1.9286	14.643	5	Reject H_0 at .01, .05, .1	σ=166.82 μ=39.471
Weibull	3.9074	2.5018	1.9286	3.7316	3	Fail to reject H_0 at .01	α=0.33444 β=3.1638

Table B.71 Goodness-of-fit and Distribution Parameters (Minnesota)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	3.1266	5	Fail to reject H_0 at .01	α_1=0.09965 α_2=1.2048
Frechet	3.9074	2.5018	1.9286	0.69938	1	Fail to reject H_0 at .01, .05, .1	α=0.36629 β=1.4305
Gamma	3.9074	2.5018	1.9286	2.1443	4	Fail to reject H_0 at .01, .05	α=0.209 β=951.32
Gumbel	3.9074	2.5018	1.9286	7.6876	6	Reject H_0 at .01, .05, .1	σ=339.1 μ=3.0954
Lognormal	3.9074	2.5018	1.9286	0.91939	2	Fail to reject H_0 at .01, .05, .1	σ=3.1297 μ=1.9772
Normal	3.9074	2.5018	1.9286	8.9995	7	Reject H_0 at .01, .05, .1	σ=434.91 μ=198.83
Weibull	3.9074	2.5018	1.9286	1.314	3	Fail to reject H_0 at .01, .05, .1	α=0.33466 β=35.613

Table B.72 Goodness-of-fit and Distribution Parameters (Mississippi)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	11.332	6	Reject H_0 at .01, .05, .1	α_1=0.05343 α_2=0.68208
Frechet	3.9074	2.5018	1.9286	2.0065	3	Fail to reject H_0 at .01, .05	α=0.40729 β=2.5707
Gamma	3.9074	2.5018	1.9286	7.0243	4	Reject H_0 at .01, .05, .1	α=0.11286 β=1395.8
Gumbel	3.9074	2.5018	1.9286	10.113	5	Reject H_0 at .01, .05, .1	σ=365.62 μ=-53.507
Lognormal	3.9074	2.5018	1.9286	0.71878	2	Fail to reject H_0 at .01, .05, .1	σ=2.7597 μ=2.4064
Normal	3.9074	2.5018	1.9286	11.547	7	Reject H_0 at .01, .05, .1	σ=468.92 μ=157.53
Weibull	3.9074	2.5018	1.9286	0.39846	1	Fail to reject H_0 at .01, .05, .1	α=0.40525 β=42.407

Table B.73 Goodness-of-fit and Distribution Parameters (Missouri)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	3.4844	5	Fail to reject H_0 at .01	α_1=0.11516 α_2=1.111
Frechet	3.9074	2.5018	1.9286	1.7583	4	Fail to reject H_0 at .01, .05, .1	α=0.3725 β=5.1655
Gamma	3.9074	2.5018	1.9286	1.1004	3	Fail to reject H_0 at .01, .05, .1	α=0.19485 β=1786.2
Gumbel	3.9074	2.5018	1.9286	7.2125	6	Reject H_0 at .01, .05, .1	σ=614.75 μ=-6.8102
Lognormal	3.9074	2.5018	1.9286	0.52576	2	Fail to reject H_0 at .01, .05, .1	σ=2.9768 μ=3.2223
Normal	3.9074	2.5018	1.9286	8.6266	7	Reject H_0 at .01, .05, .1	σ=788.45 μ=348.03
Weibull	3.9074	2.5018	1.9286	0.2844	1	Fail to reject H_0 at .01, .05, .1	α=0.3936 β=104.97

Table B.74 Goodness-of-fit and Distribution Parameters (Montana)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	21.174	7	Reject H_0 at .01, .05, .1	α_1=0.03579 α_2=0.53199
Frechet	3.9074	2.5018	1.9286	1.3023	2	Fail to reject H_0 at .01, .05, .1	α=0.43623 β=0.41609
Gamma	3.9074	2.5018	1.9286	8.6487	4	Reject H_0 at .01, .05, .1	α=0.08763 β=769.57
Gumbel	3.9074	2.5018	1.9286	12.061	5	Reject H_0 at .01, .05, .1	σ=177.62 μ=-35.091
Lognormal	3.9074	2.5018	1.9286	1.2555	1	Fail to reject H_0 at .01, .05, .1	σ=2.7399 μ=0.52298
Normal	3.9074	2.5018	1.9286	13.341	6	Reject H_0 at .01, .05, .1	σ=227.81 μ=67.434
Weibull	3.9074	2.5018	1.9286	1.9709	3	Fail to reject H_0 at .01, .05	α=0.33919 β=7.1136

Table B.75 Goodness-of-fit and Distribution Parameters (Nebraska)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	6.192	6	Reject H_0 at .01, .05, .1	α_1=0.14938 α_2=1.6631
Frechet	3.9074	2.5018	1.9286	3.1514	4	Fail to reject H_0 at .01	α=0.36531 β=2.523
Gamma	3.9074	2.5018	1.9286	0.5276	1	Fail to reject H_0 at .01, .05, .1	α=0.27375 β=414.32
Gumbel	3.9074	2.5018	1.9286	5.3805	5	Reject H_0 at .01, .05, .1	σ=169.02 μ=15.86
Lognormal	3.9074	2.5018	1.9286	1.6972	3	Fail to reject H_0 at .01, .05, .1	σ=2.893 μ=2.5221
Normal	3.9074	2.5018	1.9286	6.7444	7	Reject H_0 at .01, .05, .1	σ=216.78 μ=113.42
Weibull	3.9074	2.5018	1.9286	0.78876	2	Fail to reject H_0 at .01, .05, .1	α=0.43647 β=48.3

Table B.76 Goodness-of-fit and Distribution Parameters (Nevada)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	21.456	7	Reject H_0 at .01, .05, .1	α_1=0.0349 α_2=0.50257
Frechet	3.9074	2.5018	1.9286	1.7681	1	Fail to reject H_0 at .01, .05, .1	α=0.39933 β=0.14745
Gamma	3.9074	2.5018	1.9286	7.7399	4	Reject H_0 at .01, .05, .1	α=0.08865 β=407.82
Gumbel	3.9074	2.5018	1.9286	11.655	5	Reject H_0 at .01, .05, .1	σ=94.673 μ=-18.494
Lognormal	3.9074	2.5018	1.9286	2.4613	2	Fail to reject H_0 at .01, .05	σ=2.9473 μ=-0.39479
Normal	3.9074	2.5018	1.9286	12.986	6	Reject H_0 at .01, .05, .1	σ=121.42 μ=36.152
Weibull	3.9074	2.5018	1.9286	2.8917	3	Fail to reject H_0 at .01	α=0.31628 β=3.2109

Table B.77 Goodness-of-fit and Distribution Parameters (New Hampshire)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	N/A	N/A	N/A	α_1=0.00735 α_2=0.25716
Frechet	3.9074	2.5018	1.9286	5.1592	1	Reject H_0 at .01, .05, .1	α=0.50323 β=0.13186
Gamma	3.9074	2.5018	1.9286	27.914	6	Reject H_0 at .01, .05, .1	α=0.03555 β=921.78
Gumbel	3.9074	2.5018	1.9286	14.687	4	Reject H_0 at .01, .05, .1	σ=135.51 μ=-45.449
Lognormal	3.9074	2.5018	1.9286	6.0743	3	Reject H_0 at .01, .05, .1	σ=2.3679 μ=-0.76482
Normal	3.9074	2.5018	1.9286	15.461	5	Reject H_0 at .01, .05, .1	σ=173.8 μ=32.768
Weibull	3.9074	2.5018	1.9286	5.8441	2	Reject H_0 at .01, .05, .1	α=0.33445 β=1.7484

Table B.78 Goodness-of-fit and Distribution Parameters (New Jersey)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	5.8096	5	Reject H_0 at .01, .05, .1	α_1=0.0788 α_2=0.9146
Frechet	3.9074	2.5018	1.9286	3.3737	1	Fail to reject H_0 at .01	α=0.28967 β=0.2380
Gamma	3.9074	2.5018	1.9286	4.0263	3	Reject H_0 at .01, .05, .1	α=0.14232 β=1449.4
Gumbel	3.9074	2.5018	1.9286	8.9986	6	Reject H_0 at .01, .05, .1	σ=426.34 μ=-39.809
Lognormal	3.9074	2.5018	1.9286	4.1413	4	Reject H_0 at .01, .05, .1	σ=3.8085 μ=0.61775
Normal	3.9074	2.5018	1.9286	10.058	7	Reject H_0 at .01, .05, .1	σ=546.8 μ=206.28
Weibull	3.9074	2.5018	1.9286	3.914	2	Reject H_0 at .01, .05, .1	α=0.26186 β=13.55

Table B.79 Goodness-of-fit and Distribution Parameters (New Mexico)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	34.55	7	Reject H_0 at .01, .05, .1	α_1=0.02345 α_2=0.54466
Frechet	3.9074	2.5018	1.9286	2.2373	1	Fail to reject H_0 at .01, .05	α=0.43397 β=0.24648
Gamma	3.9074	2.5018	1.9286	18.142	6	Reject H_0 at .01, .05, .1	α=0.0562 β=509.13
Gumbel	3.9074	2.5018	1.9286	12.213	4	Reject H_0 at .01, .05, .1	σ=94.108 μ=-25.707
Lognormal	3.9074	2.5018	1.9286	2.5867	3	Fail to reject H_0 at .01	σ=2.6848 μ=0.00923
Normal	3.9074	2.5018	1.9286	13.114	5	Reject H_0 at .01, .05, .1	σ=120.7 μ=28.614
Weibull	3.9074	2.5018	1.9286	2.4789	2	Fail to reject H_0 at .01, .05	α=0.35888 β=4.1009

Table B.80 Goodness-of-fit and Distribution Parameters (New York)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	67.493	7	Reject H_0 at .01, .05, .1	α_1=0.02134 α_2=0.51224
Frechet	3.9074	2.5018	1.9286	3.2855	3	Fail to reject H_0 at .01	α=0.34778 β=2.3495
Gamma	3.9074	2.5018	1.9286	19.694	6	Reject H_0 at .01, .05, .1	α=0.05315 β=5996.3
Gumbel	3.9074	2.5018	1.9286	12.653	4	Reject H_0 at .01, .05, .1	σ=1077.8 μ=-303.45
Lognormal	3.9074	2.5018	1.9286	1.7773	2	Fail to reject H_0 at .01, .05, .1	σ=3.1192 μ=2.5743
Normal	3.9074	2.5018	1.9286	13.565	5	Reject H_0 at .01, .05, .1	σ=1382.4 μ=318.69
Weibull	3.9074	2.5018	1.9286	0.89204	1	Fail to reject H_0 at .01, .05, .1	α=0.36976 β=58.34

Table B.81 Goodness-of-fit and Distribution Parameters (North Carolina)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	54.492	7	Reject H_0 at .01, .05, .1	α_1=0.03147 α_2=0.46144
Frechet	3.9074	2.5018	1.9286	2.6804	3	Fail to reject H_0 at .01	α=0.39007 β=1.4945
Gamma	3.9074	2.5018	1.9286	10.156	4	Reject H_0 at .01, .05, .1	α=0.08472 β=1919.8
Gumbel	3.9074	2.5018	1.9286	11.811	5	Reject H_0 at .01, .05, .1	σ=435.68 μ=-88.841
Lognormal	3.9074	2.5018	1.9286	1.3058	2	Fail to reject H_0 at .01, .05, .1	σ=2.8891 μ=1.9369
Normal	3.9074	2.5018	1.9286	13.193	6	Reject H_0 at .01, .05, .1	σ=558.78 μ=162.64
Weibull	3.9074	2.5018	1.9286	0.93178	1	Fail to reject H_0 at .01, .05, .1	α=0.37056 β=28.82

Table B.82 Goodness-of-fit and Distribution Parameters (North Dakota)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	23.108	7	Reject H_0 at .01, .05, .1	α_1=0.03153 α_2=0.62075
Frechet	3.9074	2.5018	1.9286	2.1541	1	Fail to reject H_0 at .01, .05	α=0.33741 β=0.35109
Gamma	3.9074	2.5018	1.9286	9.916	4	Reject H_0 at .01, .05, .1	α=0.06972 β=2049.4
Gumbel	3.9074	2.5018	1.9286	11.794	5	Reject H_0 at .01, .05, .1	σ=421.94 μ=-100.66
Lognormal	3.9074	2.5018	1.9286	2.609	3	Fail to reject H_0 at .01	σ=3.3834 μ=0.73847
Normal	3.9074	2.5018	1.9286	12.801	6	Reject H_0 at .01, .05, .1	σ=541.15 μ=142.89
Weibull	3.9074	2.5018	1.9286	2.4584	2	Fail to reject H_0 at .01, .05	α=0.29678 β=12.057

Table B.83 Goodness-of-fit and Distribution Parameters (Ohio)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	7.463	5	Reject H_0 at .01, .05, .1	α_1=0.10483 α_2=0.85078
Frechet	3.9074	2.5018	1.9286	3.667	4	Fail to reject H_0 at .01	α=0.36124 β=3.1683
Gamma	3.9074	2.5018	1.9286	2.3073	3	Fail to reject H_0 at .01, .05	α=0.19903 β=708.47
Gumbel	3.9074	2.5018	1.9286	7.5028	6	Reject H_0 at .01, .05, .1	σ=246.43 μ=-1.2424
Lognormal	3.9074	2.5018	1.9286	1.6651	2	Fail to reject H_0 at .01, .05, .1	σ=2.8786 μ=2.7699
Normal	3.9074	2.5018	1.9286	9.1088	7	Reject H_0 at .01, .05, .1	σ=316.06 μ=141.0
Weibull	3.9074	2.5018	1.9286	0.59744	1	Fail to reject H_0 at .01, .05, .1	α=0.45089 β=58.594

Table B.84 Goodness-of-fit and Distribution Parameters (Oklahoma)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	2.0054	4	Fail to reject H_0 at .01, .05	α_1=0.13583 α_2=1.2035
Frechet	3.9074	2.5018	1.9286	2.1681	5	Fail to reject H_0 at .01, .05	α=0.34661 β=1.453
Gamma	3.9074	2.5018	1.9286	0.96555	1	Fail to reject H_0 at .01, .05, .1	α=0.21808 β=647.5
Gumbel	3.9074	2.5018	1.9286	6.681	6	Reject H_0 at .01, .05, .1	σ=235.76 μ=5.1206
Lognormal	3.9074	2.5018	1.9286	1.412	3	Fail to reject H_0 at .01, .05, .1	σ=3.1717 μ=2.0678
Normal	3.9074	2.5018	1.9286	7.887	7	Reject H_0 at .01, .05, .1	σ=302.37 μ=141.2
Weibull	3.9074	2.5018	1.9286	0.96766	2	Fail to reject H_0 at .01, .05, .1	α=0.36467 β=37.261

Table B.85 Goodness-of-fit and Distribution Parameters (Oregon)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	19.051	7	Reject H_0 at .01, .05, .1	α_1=0.03998 α_2=0.51667
Frechet	3.9074	2.5018	1.9286	1.7682	3	Fail to reject H_0 at .01, .05, .1	α=0.36205 β=1.455
Gamma	3.9074	2.5018	1.9286	6.0362	4	Reject H_0 at .01, .05, .1	α=0.09995 β=2647.5
Gumbel	3.9074	2.5018	1.9286	11.289	5	Reject H_0 at .01, .05, .1	σ=652.6 μ=-112.08
Lognormal	3.9074	2.5018	1.9286	0.70042	2	Fail to reject H_0 at .01, .05, .1	σ=3.1612 μ=2.0246
Normal	3.9074	2.5018	1.9286	12.688	6	Reject H_0 at .01, .05, .1	σ=836.99 μ=264.61
Weibull	3.9074	2.5018	1.9286	0.6982	1	Fail to reject H_0 at .01, .05, .1	α=0.33837 β=36.15

Table B.86 Goodness-of-fit and Distribution Parameters (Pennsylvania)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	48.247	7	Reject H_0 at .01, .05, .1	α_1=0.01217 α_2=0.33168
Frechet	3.9074	2.5018	1.9286	1.5408	3	Fail to reject H_0 at .01, .05, .1	α=0.35102 β=1.9519
Gamma	3.9074	2.5018	1.9286	20.634	6	Reject H_0 at .01, .05, .1	α=0.04102 β=22789.0
Gumbel	3.9074	2.5018	1.9286	13.832	4	Reject H_0 at .01, .05, .1	σ=3598.8 μ=-1142.4
Lognormal	3.9074	2.5018	1.9286	0.62134	1	Fail to reject H_0 at .01, .05, .1	σ=3.3377 μ=2.4029
Normal	3.9074	2.5018	1.9286	14.639	5	Reject H_0 at .01, .05, .1	σ=4615.6 μ=934.83
Weibull	3.9074	2.5018	1.9286	0.6899	2	Fail to reject H_0 at .01, .05, .1	α=0.30622 β=59.187

Table B.87 Goodness-of-fit and Distribution Parameters (Rhode Island)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	78.475	7	Reject H_0 at .01, .05, .1	α_1=0.00517 α_2=0.16262
Frechet	3.9074	2.5018	1.9286	10.621	1	Reject H_0 at .01, .05, .1	α=0.58813 β=0.05186
Gamma	3.9074	2.5018	1.9286	32.012	6	Reject H_0 at .01, .05, .1	α=0.03093 β=766.62
Gumbel	3.9074	2.5018	1.9286	15.424	4	Reject H_0 at .01, .05, .1	σ=105.12 μ=-36.967
Lognormal	3.9074	2.5018	1.9286	11.461	2	Reject H_0 at .01, .05, .1	σ=1.8918 μ=-1.8347
Normal	3.9074	2.5018	1.9286	16.135	5	Reject H_0 at .01, .05, .1	σ=134.83 μ=23.712
Weibull	3.9074	2.5018	1.9286	12.021	3	Reject H_0 at .01, .05, .1	α=0.3223 β=0.51817

Table B.88 Goodness-of-fit and Distribution Parameters (South Carolina)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	28.975	7	Reject H_0 at .01, .05, .1	α_1=0.03277 α_2=0.68169
Frechet	3.9074	2.5018	1.9286	1.7163	3	Fail to reject H_0 at .01, .05, .1	α=0.52063 β=0.65812
Gamma	3.9074	2.5018	1.9286	18.236	6	Reject H_0 at .01, .05, .1	α=0.06889 β=340.18
Gumbel	3.9074	2.5018	1.9286	11.521	4	Reject H_0 at .01, .05, .1	σ=69.616 μ=-16.749
Lognormal	3.9074	2.5018	1.9286	0.77265	1	Fail to reject H_0 at .01, .05, .1	σ=2.2567 μ=0.75685
Normal	3.9074	2.5018	1.9286	12.444	5	Reject H_0 at .01, .05, .1	σ=89.286 μ=23.435
Weibull	3.9074	2.5018	1.9286	0.83799	2	Fail to reject H_0 at .01, .05, .1	α=0.44466 β=6.6648

Table B.89 Goodness-of-fit and Distribution Parameters (South Dakota)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.1177	1	Reject H_0 at .01, .05, .1	α_1=0.04464 α_2=0.82388
Frechet	3.9074	2.5018	1.9286	10.386	3	Reject H_0 at .01, .05, .1	α=0.30783 β=0.25174
Gamma	3.9074	2.5018	1.9286	8.1661	2	Reject H_0 at .01, .05, .1	α=0.09901 β=1088.5
Gumbel	3.9074	2.5018	1.9286	10.524	4	Reject H_0 at .01, .05, .1	σ=267.06 μ=-46.376
Lognormal	3.9074	2.5018	1.9286	17.671	6	Reject H_0 at .01, .05, .1	σ=2.7839 μ=1.1993
Normal	3.9074	2.5018	1.9286	11.56	5	Reject H_0 at .01, .05, .1	σ=342.51 μ=107.77
Weibull	3.9074	2.5018	1.9286	20.646	7	Reject H_0 at .01, .05, .1	α=0.33971 β=30.213

Table B.90 Goodness-of-fit and Distribution Parameters (Tennessee)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	4.9121	5	Reject H_0 at .01, .05, .1	α_1=0.10145 α_2=0.72314
Frechet	3.9074	2.5018	1.9286	2.1189	4	Fail to reject H_0 at .01, .05	α=0.42831 β=2.5053
Gamma	3.9074	2.5018	1.9286	1.6445	3	Fail to reject H_0 at .01, .05, .1	α=0.21182 β=408.49
Gumbel	3.9074	2.5018	1.9286	7.2775	6	Reject H_0 at .01, .05, .1	σ=146.58 μ=1.9141
Lognormal	3.9074	2.5018	1.9286	0.74398	2	Fail to reject H_0 at .01, .05, .1	σ=2.5857 μ=2.292
Normal	3.9074	2.5018	1.9286	9.106	7	Reject H_0 at .01, .05, .1	σ=188.0 μ=86.524
Weibull	3.9074	2.5018	1.9286	0.353	1	Fail to reject H_0 at .01, .05, .1	α=0.45004 β=34.267

Table B.91 Goodness-of-fit and Distribution Parameters (Texas)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	7.2542	6	Reject H_0 at .01, .05, .1	α_1=0.12425 α_2=1.5681
Frechet	3.9074	2.5018	1.9286	5.1349	4	Reject H_0 at .01, .05, .1	α=0.33115 β=14.935
Gamma	3.9074	2.5018	1.9286	2.95	3	Fail to reject H_0 at .01	α=0.20094 β=3249.3
Gumbel	3.9074	2.5018	1.9286	6.9637	5	Reject H_0 at .01, .05, .1	σ=1135.7 μ=-2.6089
Lognormal	3.9074	2.5018	1.9286	2.6114	2	Fail to reject H_0 at .01	σ=2.9776 μ=4.4569
Normal	3.9074	2.5018	1.9286	8.2946	7	Reject H_0 at .01, .05, .1	σ=1456.6 μ=652.92
Weibull	3.9074	2.5018	1.9286	0.87506	1	Fail to reject H_0 at .01, .05, .1	α=0.47334 β=314.63

Table B.92 Goodness-of-fit and Distribution Parameters (Utah)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	41.116	7	Reject H_0 at .01, .05, .1	α_1=0.01758 α_2=0.30004
Frechet	3.9074	2.5018	1.9286	1.3389	2	Fail to reject H_0 at .01, .05, .1	α=0.45901 β=0.50512
Gamma	3.9074	2.5018	1.9286	15.171	6	Reject H_0 at .01, .05, .1	α=0.06427 β=892.36
Gumbel	3.9074	2.5018	1.9286	13.338	4	Reject H_0 at .01, .05, .1	σ=176.39 μ=-44.462
Lognormal	3.9074	2.5018	1.9286	1.0287	1	Fail to reject H_0 at .01, .05, .1	σ=2.5913 μ=0.65106
Normal	3.9074	2.5018	1.9286	14.707	5	Reject H_0 at .01, .05, .1	σ=226.23 μ=57.353
Weibull	3.9074	2.5018	1.9286	1.3923	3	Fail to reject H_0 at .01, .05, .1	α=0.36567 β=7.3315

Table B.93 Goodness-of-fit and Distribution Parameters (Vermont)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	52.73	7	Reject H_0 at .01, .05, .1	α_1=0.01194 α_2=0.24812
Frechet	3.9074	2.5018	1.9286	3.7022	1	Fail to reject H_0 at .01	α=0.44507 β=0.11989
Gamma	3.9074	2.5018	1.9286	17.815	6	Reject H_0 at .01, .05, .1	α=0.05046 β=887.64
Gumbel	3.9074	2.5018	1.9286	14.476	4	Reject H_0 at .01, .05, .1	σ=155.47 μ=-44.947
Lognormal	3.9074	2.5018	1.9286	4.3998	2	Reject H_0 at .01, .05, .1	σ=2.6562 μ=-0.71951
Normal	3.9074	2.5018	1.9286	15.673	5	Reject H_0 at .01, .05, .1	σ=199.4 μ=44.792
Weibull	3.9074	2.5018	1.9286	4.7085	3	Reject H_0 at .01, .05, .1	α=0.31447 β=2.0776

Table B.94 Goodness-of-fit and Distribution Parameters (Virginia)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	7.5798	5	Reject H_0 at .01, .05, .1	α_1=0.06149 α_2=0.43206
Frechet	3.9074	2.5018	1.9286	1.2165	1	Fail to reject H_0 at .01, .05, .1	α=0.33639 β=0.82306
Gamma	3.9074	2.5018	1.9286	2.6218	4	Fail to reject H_0 at .01	α=0.17599 β=1383.4
Gumbel	3.9074	2.5018	1.9286	9.4263	6	Reject H_0 at .01, .05, .1	σ=452.52 μ=-17.722
Lognormal	3.9074	2.5018	1.9286	1.3179	2	Fail to reject H_0 at .01, .05, .1	σ=3.4068 μ=1.5682
Normal	3.9074	2.5018	1.9286	11.282	7	Reject H_0 at .01, .05, .1	σ=580.37 μ=243.48
Weibull	3.9074	2.5018	1.9286	1.5662	3	Fail to reject H_0 at .01, .05, .1	α=0.30134 β=27.55

Table B.95 Goodness-of-fit and Distribution Parameters (Washington)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	1.4545	3	Fail to reject H_0 at .01, .05, .1	α_1=0.1736 α_2=0.86217
Frechet	3.9074	2.5018	1.9286	2.1427	5	Fail to reject H_0 at .01, .05	α=0.37351 β=1.2695
Gamma	3.9074	2.5018	1.9286	1.7603	4	Fail to reject H_0 at .01, .05, .1	α=0.35478 β=170.46
Gumbel	3.9074	2.5018	1.9286	5.1396	6	Reject H_0 at .01, .05, .1	σ=79.162 μ=14.781
Lognormal	3.9074	2.5018	1.9286	1.4353	2	Fail to reject H_0 at .01, .05, .1	σ=2.8871 μ=1.792
Normal	3.9074	2.5018	1.9286	6.9479	7	Reject H_0 at .01, .05, .1	σ=101.53 μ=60.474
Weibull	3.9074	2.5018	1.9286	0.98067	1	Fail to reject H_0 at .01, .05, .1	α=0.41908 β=24.036

Table B.96 Goodness-of-fit and Distribution Parameters (West Virginia)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	2.394	5	Fail to reject H_0 at .01, .05	α_1=0.17334 α_2=1.5005
Frechet	3.9074	2.5018	1.9286	2.1727	4	Fail to reject H_0 at .01, .05	α=0.37248 β=2.4848
Gamma	3.9074	2.5018	1.9286	0.5019	2	Fail to reject H_0 at .01, .05, .1	α=0.25464 β=423.49
Gumbel	3.9074	2.5018	1.9286	5.7509	6	Reject H_0 at .01, .05, .1	σ=166.62 μ=11.66
Lognormal	3.9074	2.5018	1.9286	0.69275	3	Fail to reject H_0 at .01, .05, .1	σ=2.8608 μ=2.4794
Normal	3.9074	2.5018	1.9286	6.9397	7	Reject H_0 at .01, .05, .1	σ=213.7 μ=107.84
Weibull	3.9074	2.5018	1.9286	0.22348	1	Fail to reject H_0 at .01, .05, .1	α=0.44072 β=44.712

Table B.97 Goodness-of-fit and Distribution Parameters (Wisconsin)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	7.2542	5	Reject H_0 at .01, .05, .1	α_1=0.07984 α_2=1.1042
Frechet	3.9074	2.5018	1.9286	2.3455	3	Fail to reject H_0 at .01, .05	α=0.36293 β=0.58021
Gamma	3.9074	2.5018	1.9286	3.9128	4	Reject H_0 at .01, .05, .1	α=0.13102 β=487.58
Gumbel	3.9074	2.5018	1.9286	8.545	6	Reject H_0 at .01, .05, .1	σ=137.61 μ=-15.546
Lognormal	3.9074	2.5018	1.9286	2.3259	2	Fail to reject H_0 at .01, .05	σ=3.0542 μ=1.0924
Normal	3.9074	2.5018	1.9286	9.6262	7	Reject H_0 at .01, .05, .1	σ=176.49 μ=63.883
Weibull	3.9074	2.5018	1.9286	1.8844	1	Fail to reject H_0 at .01, .05, .1	α=0.35561 β=13.864

Table B.98 Goodness-of-fit and Distribution Parameters (Wyoming)

Distribution	α =0.01	α =0.05	α = 0.1	Anderson Darling	Rank	Reject/Accept	Parameters
Beta	3.9074	2.5018	1.9286	19.972	7	Reject H_0 at .01, .05, .1	α_1=0.04447 α_2=0.50152
Frechet	3.9074	2.5018	1.9286	2.7093	1	Fail to reject H_0 at .01	α=0.52568 β=0.12718
Gamma	3.9074	2.5018	1.9286	8.7977	4	Reject H_0 at .01, .05, .1	α=0.11439 β=52.889
Gumbel	3.9074	2.5018	1.9286	10.128	5	Reject H_0 at .01, .05, .1	σ=13.947 μ=-2.0005
Lognormal	3.9074	2.5018	1.9286	3.7711	2	Fail to reject H_0 at .01	σ=2.2137 μ=-0.90321
Normal	3.9074	2.5018	1.9286	11.536	6	Reject H_0 at .01, .05, .1	σ=17.888 μ=6.05
Weibull	3.9074	2.5018	1.9286	4.041	3	Reject H_0 at .01, .05, .1	α=0.40496 β=1.3386

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